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SELF-SIMILAR ANALYSIS OF A SYSTEM OF EQUATIONS GIVEN BY A DIFFUSION PROCESS

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Abstract. This paper investigates the self-similar analysis of a system of equations given by a diffusion process. The study focuses on a coupled system of nonlinear parabolic partial differential equations (PDEs) that describe mass conservation laws and filtration flows in heterogeneous media. To evaluate the qualitative properties of the solutions, a system of self-similar equations and the nonlinear splitting method were used. The findings contribute to understanding nonlinear filtration dynamics in various physical and engineering applications, such as groundwater flow, oil extraction, and heat transfer in porous media. Additionally, the study provides insights into the mathematical properties of such systems, including regularity, stability, and the qualitative behavior of solutions in multidimensional settings.

Keywords: mathematical modeling, numerical solutions, asymptotic behavior, non-homogeneous media, self-similar systems of equations.

Diffuziyali jarayon bilan berilgan tenglamalar sistemasini avtomodel tahlili

Annotatsiya. Ushbu maqolada diffuziyali jarayon bilan berilgan tenglamalar sistemasining avtomodel tahlili o‘rganilgan. Tadqiqotda asosiy e’tibor bir-jinsli bo‘lmagan muhitda massa saqlanish qonunlari va filtrlash oqimlarini tavsiflovchi chiziqli bo‘lmagan parabolik qisman differensial tenglamalarning birlashtirilgan tizimiga qaratilgan. Yechimlarning sifat xususiyatlarini baholash uchun avtomodel tenglamalar sistemasi va chiziqli bo‘lmagan bo‘linish usuli qo‘llanilgan. Topilmalar yer osti suvlari oqimi, neft qazib olish va g‘ovakli muhitda issiqliq uzatish kabi turli xil tabiiy va muhandislik dasturlarida chiziqli bo‘lmagan filtrlash dinamikasini tushunishga yordam beradi. Bundan tashqari, tadqiqot bunday tizimlarning matematik xususiyatlari, jumladan, ko‘p o‘lchovli sohalarda yechimlarning barqarorligi va sifatli xatti-harakatlari haqida tushuncha beradi.

Kalit so‘zlar: matematik model, avtomodel tenglamalar sistemasi, sonli yechimlar, asimpotik yechim, bir-jinsli bo’lmagan muhit.

Автомодельный анализ системы уравнений, заданной процессом диффузии

Аннотация. В данной статье исследуется автомоделные анализ системы уравнений, заданной процессом диффузии. Исследование сосредоточено на связанной системе нелинейных параболических уравнений в частных производных (PDE), которые описывают законы сохранения массы и фильтрационные потоки в неоднородных средах. Для оценки качественных свойств решений использовались

система самоподобных уравнений и метод нелинейного расщепления. Результаты способствуют пониманию динамики нелинейной фильтрации в различных физических и инженерных приложениях, таких как поток грунтовых вод, добыча нефти и перенос тепла в пористых средах. Кроме того, исследование дает представление о математических свойствах таких систем, включая регулярность, устойчивость и качественное поведение решений в многомерных условиях.

Ключевые слова: математическое моделирование, автомодельные системы уравнений, численные решения, асимптотическое поведение, гетерогенные среды.

Introduction. Consider in domain $¤_t = \{(t,x): t>0,\ x \in R_+^2\}$ the properties of solutions of double nonlinearity of the Cauchy problem for the following system of diffusion equations

$$\begin{cases} \rho_1(x)u_t = div\left(\rho_2(x)u^{m_1-1}\left|\nabla u^{k_1}\right|^{p-2}\nabla u^{l_1}\right)+\rho_3(x)\varepsilon_1 v^{k_1} \\ \\ \rho_1(x)v_t = div\left(\rho_2(x)v^{m_2-1}\left|\nabla v^{k_2}\right|^{p-2}\nabla v^{l_2}\right)+\rho_3(x)\varepsilon_2 u^{k_2} \end{cases} \tag{1}$$

$$\begin{cases} u(0,x)=u_0(x)\geq 0 \\ \\ v(0,x)=v_0(x)\geq 0,\ x\in R_+^2 \end{cases} \tag{2}$$

where

$k_1,k_2 \geq 1, k \geq 0$ $m_1,m_2 \geq 1,\ p\geq 2,\ \rho_1(x)=|x|^l,\ \rho_2(x)=|x|^n,\ \rho_3(\chi)=|x|^k, \varepsilon_1,\varepsilon_1 \geq 1,\quad l_1,l_2 \geq 1$ are given parameters, $\nabla(.)-\underset{x}{grad}(.)$.

From the perspective of mathematical modeling, it is reasonable to consider a weak solution that satisfies the necessary conditions and possesses properties of boundedness and continuity in the

$$0\leq u(t,x),\ v(t,x), v^{m_1-1}\left|\nabla u^{k_1}\right|^{p-2}\nabla u^{l_1}, u^{m_2-1}\left|\nabla v^{k_2}\right|^{p-2}v^{l_2} \in C(¤_t), \tag{3}$$

similarly, system (1) is considered in the context of the integral equality [6-7]:

$$\begin{cases} \int\limits_0^t\int\limits_\Omega [u\frac{\partial \psi}{\partial t}+v^{m_1-1}\left|\nabla u^{k_1}\right|^{p-2}\nabla u^{l_1}\nabla\psi-\int\limits_\Omega \psi u_0]dxdt=0, \\ \int\limits_0^t\int\limits_\Omega [v\frac{\partial \psi}{\partial t}+u^{m_2-1}\left|\nabla v^{k_2}\right|^{p-2}\nabla v^{l_2}\nabla\psi-\int\limits_\Omega \psi u_0]dxdt=0, \end{cases} \tag{4}$$

for a sufficiently smooth function $\psi(t,x)$ with compact support, as used in mathematical modeling theory.

In works [11, 14, 16] for the following equations with variable parameters:

$$\rho(x)\frac{\partial u}{\partial t}=\nabla(u^{m-1}\left|\nabla u^k\right|^{p-2}\nabla u)+u^\beta,\quad x\in R_+^2,\ t>0,$$

$$\rho(x)\frac{\partial u}{\partial t}=\nabla(u^{m-1}\left|\nabla u^{k}\right|^{p-2}\nabla u)+\rho(x)u^{\beta},\quad x\in R_{+}^{2},\ t>0$$

here $p>1,\ \ p+m-3>0,\ \ \beta>m-2+p,\ \ \rho(x)=\left|x\right|^{k}$ under finite parameters and certain constraints on the initial function, it is demonstrated that any non-trivial solution to the Cauchy problem exhibits blow-up within a finite time with bounded velocity. Furthermore, the authors derived a universal estimate for the solution in the vicinity of the smoothing point.

To initiate the process of finding a self-similar solution, it is necessary first to determine the form in which the solution will be sought.

Main results. To achieve this, we seek the solutions of system (1) in the form of

$$\begin{cases}u(t,x)=(T+t)^{-\alpha_1}w(\tau(t),x),\\ v(t,x)=(T+t)^{-\alpha_2}z(\tau(t),x),\end{cases}\tag{5}$$

where α_1,α_2 represents the coefficients to be determined, and the unknown function $\tau(t)$, upon substituting (5) into (1) and performing straightforward calculations, is defined as follows:

$$\tau(t)=\int(T+t)^{(m_2-1)\alpha_1+k_1k_2l_1l_2(p-2)\alpha_2}dt=\int(T+t)^{(m_1-1)\alpha_2+k_1k_2l_1l_2(p-2)\alpha_1}dt.$$

For $1-(m_1-1)\alpha_2+k_1k_2l_1l_2(p-2)\alpha_1>0$, the evaluation of the integral for $\tau(t)$ yields the following result

$$\tau(t)=(T+t)^{p_1}\ /\ p_1,\quad p_1=1-(m_1-1)\alpha_2+k_1k_2l_1l_2(p-2)\alpha_1.$$

As a result, a new system will be obtained

$$\begin{cases}\dfrac{\partial w}{\partial\tau}=\nabla\left(z^{m_1-1}\left|\nabla w^{k_1}\right|^{p-2}\nabla w^{l_1}\right)+\dfrac{\alpha_1}{(1-\alpha_2(m_1-1)+k_1k_2l_1l_2\alpha_1(p-2))\tau(t)}w,\\ \dfrac{\partial z}{\partial\tau}=\nabla\left(w^{m_2-1}\left|\nabla z^{k_2}\right|^{p-2}\nabla z^{l_2}\right)+\dfrac{\alpha_2}{(1-(m_2-1)\alpha_1+k_1k_2l_1l_2(p-2)\alpha_2)\tau(t)}z,\end{cases}\tag{6}$$

Observe that, when calculating the integral at $\left(m_2-1\right)\alpha_1+k_1k_2l_1l_2\left(p-2\right)\alpha_2\neq1$ the following expression is obtained for $\tau(t)$

$$\tau(t)=T+t\tag{7}$$

In the case where $\left(m_2-1\right)\alpha_1+k_1k_2l_1l_2\left(p-2\right)\alpha_2>1$, considering the expression for $\tau(t)$, is rewritten according to equation (6), it becomes as follows:

$$\begin{cases}\dfrac{\partial w}{\partial\tau}=div\left(z^{m_1-1}\left|\nabla w^{k_1}\right|^{p-2}\nabla w^{l_1}\right)+(b_1/\tau)w,\\ \dfrac{\partial z}{\partial t}=div\left(w^{m_2-1}\left|\nabla z^{k_2}\right|^{p-2}\nabla z^{l_2}\right)+(b_2/\tau)z,\end{cases}\tag{8}$$

where

$$\begin{cases} b_1 = \dfrac{\alpha_1}{1-(m_1-1)\alpha_2 + k_1k_2l_1l_2(p-2)\alpha_1}, \\ b_2 = \dfrac{\alpha_2}{1-(m_1-1)\alpha_1 + k_1k_2l_1l_2(p-2)\alpha_2}. \end{cases}$$

Subsequently, a new transformation of the shape is introduced

$$\begin{cases} w(\tau(t),x) = f(\zeta), \\ z(\tau(t),x) = \psi(\zeta), \quad \xi = \dfrac{|x|}{[\tau(t)]^{1/p}}, \end{cases} \tag{9}$$

The calculation procedure in equation (6) is carried out by substituting equation (9)

$$\alpha_2(m_1-1) + k_1k_2l_1l_2(-2+p)\alpha_1 = (-1+m_2)\alpha_1 + k_1k_2l_1l_2(-2+p)\alpha_2 \tag{10}$$

for the function $f(\xi),\ \psi(\xi)$ the following new self-similar system of equations is derived

$$\begin{cases} \zeta^{1-N}\dfrac{d}{d\xi}\left(\zeta^{N-1}\psi^{m_1-1}\left|\dfrac{df^k}{d\zeta}\right|^{p-2}\dfrac{df^l}{d\zeta}\right) + \dfrac{\xi}{p}\dfrac{df}{d\zeta} + b_1 f = 0, \\ \zeta^{1-N}\dfrac{d}{d\xi}\left(\zeta^{N-1}f^{m_2-1}\left|\dfrac{d\psi^k}{d\zeta}\right|^{p-2}\dfrac{d\psi^l}{d\zeta}\right) + \dfrac{\xi}{p}\dfrac{d\psi}{d\zeta} + b_2\psi = 0, \end{cases} \tag{11}$$

were

$$\begin{cases} b_1 = \dfrac{\alpha_1}{1-(-1+m_1)\alpha_2 - kl\alpha_1(-2+p)}, \\ b_2 = \dfrac{\alpha_2}{1-(-1+m_2)\alpha_1 - kl\alpha_2(-2+p)}. \end{cases}$$

Next, we determine the values of α_1, α_2 from the solution of the system of algebraic equations (10) and from the relation

$$\begin{cases} \dfrac{\alpha_1}{1-(m_1-1)\alpha_2 + k_1k_2l_1l_2(p-2)\alpha_1} = N/p, \\ \dfrac{\alpha_2}{1-(m_2-1)\alpha_1 + k_1k_2l_1l_2(p-2)\alpha_2} = N/p. \end{cases} \tag{12}$$

where

$$\alpha_1 = \frac{m_1+1-p}{m_2+1-p}\alpha_2,$$

$$\begin{cases} \alpha_2 = \dfrac{(m_2+1-p)N}{(m_2+1-p)(p+k_1k_2l_1l_2(p-2)N + (m_2-1)(m_1+1-p)N)} \\ \alpha_1 = \dfrac{(m_1+1-p)N}{(m_2+1-p)(p+k_1k_2l_1l_2(p-2)N) + (m_2-1)(m_1+1-p)N)} \end{cases} \tag{13}$$

Now, let us calculate $p_1 = 1-(m_1-1)\alpha_2 + k_1k_2l_1l_2(p-2)\alpha_1$.

Taking into account equation (13), we obtain

$$p_1 = 1 - \frac{[(m_1-1)(m_2+1-p)+k_1k_2l_1l_2(p-2)(m_1+1-p)]N}{(m_2+1-p)(p+k_1k_2l_1l_2(p-2)N)+(m_2-1)(m_1+1-p)N)} > 0 \qquad (14)$$

i.e.

$$\tau(t) = \frac{(T+t)^{p_1}}{p_1}.$$

Note that the functions

$$\begin{cases} f(\xi) = A\overline{f}(\xi) \\ \psi(\xi) = B\overline{\psi}(\xi) \end{cases} \qquad (15)$$

where *A* and *B* are the roots of the algebraic system for

$$\begin{cases} (\gamma\gamma_1)^{p-1} A^{kl(p-2)} B^{m_1-1} = \dfrac{1}{p}, \\ (\gamma\gamma_2)^{p-1} A^{m_1-1} B^{kl(p-2)} = \dfrac{1}{p}, \end{cases} \qquad (16)$$

satisfy in region $|\xi| < a^{(p-1)/p}$ the self-similar system

$$\begin{cases} \xi^{1-N}\dfrac{d}{d\xi}\left(\xi^{N-1}\psi^{m_1-1}\left|\dfrac{df^{k_1}}{d\xi}\right|^{p-2}\dfrac{df^{l_1}}{d\xi}\right)+\dfrac{\xi}{p}\dfrac{df}{d\xi}+\dfrac{N}{p}f=0, \\ \xi^{1-N}\dfrac{d}{d\xi}\left(\xi^{N-1}f^{m_2-1}\left|\dfrac{d\psi^{k_2}}{d\xi}\right|^{p-2}\dfrac{d\psi^{l_2}}{d\xi}\right)+\dfrac{\xi}{p}\dfrac{d\psi}{d\xi}+\dfrac{N}{p}\psi=0 \end{cases} \qquad (17)$$

in the classical sense. Continuing with zero of the function $f(\xi) = A\overline{f}(\xi),\ \psi(\xi) = B\overline{\psi}(\xi),$ for the value $|\xi| \geq a^{(p-1)/p}$ we obtain the following Zel'dovich-Companies type solution for system (1)

$$\begin{cases} u(t,x) = A(T+t)^{-\alpha_1}\overline{f}(\xi), \\ v(t,x) = B(T+t)^{-\alpha_2}\overline{\psi}(\xi), \end{cases} \qquad (18)$$

where constants $A,\ B,\ \alpha_1,\ \alpha_2$ and functions $\overline{f}(\xi),\ \overline{\psi}(\xi)$ were defined above. Therefore, the solution (6) constructed by us with $\alpha_1,\ \alpha_2$ given by formula (13) for $p_1 > 0$ is a solution of the Zel'dovich - Companies type for system (1).

Therefore, the solution we have constructed above reduces to the following solution of Zel'dovich - Companies and Martinson-Pavlov for

$$u(t,x) = (T+t)^{-\alpha}\overline{f}(\xi),$$

$$\alpha = \frac{N}{p+(p+m-3)N}, \quad \xi = \frac{|x|}{[\tau(t)]^{1/p}}$$

$$\tau(t) = \frac{[p+(p+m-3)N](T+t)^{\frac{p}{p+(p+m-3)N}}}{p},$$

$$\overline{f}(\xi) = (a - \xi^{p/(p-1)})_+^{1/(m-1)},$$

previously known for the case of a single equation (12-18).

We note that solutions of the Zel'dovich-Companies and Martinson-Pavlov types are widely used for comparing solutions, proving the global solvability and insolvability of the Cauchy problem, and analyzing the properties of finite velocity diffusion of the solution, for example, for an equation of the form [3-5]

$$\frac{\partial u}{\partial t} = div\left(u^{m_1-1}|\nabla u|^{p-2}\nabla u\right) + \varepsilon u^{\beta}, \quad \varepsilon = \pm 1.$$

and also, for systems of such equations [1, 2, 12, 19, 20].

Now, we proceed with the proof of the finite velocity of diffusion property for the system (1). To establish this property, we introduce the functions

$$\begin{cases} u_+(t,x) = (T+t)^{-\alpha_1}\overline{f}(\xi), \\ v_+(t,x) = (T+t)^{-\alpha_2}\overline{\psi}(\xi), \end{cases} \Rightarrow \xi = |x|/[\tau(t)]^{1/p}$$

$$\begin{cases} \alpha_1 = \dfrac{(m_1+1-p)N}{(m_2+1-p)(p+k_1k_2l_1l_2(p-2)N)+(m_2-1)(m_1+1-p)N)}, \\ \alpha_2 = \dfrac{(m_2+1-p)N}{(m_2+1-p)(p+k_1k_2l_1l_2(p-2)N+(m_2-1)(m_1+1-p)N)}, \end{cases}$$

$$\begin{cases} \overline{f}(\xi) = \left(a-\xi^{\gamma}\right)_+^{\gamma_1}, \ \overline{\psi}(\xi) = \left(a-\xi^{\gamma}\right)_+^{\gamma_2}, \\ \overline{f}(\xi) = \left(a+\xi^{\gamma}\right)^{\gamma_1}, \ \overline{\psi}(\xi) = \left(a+\xi^{\gamma}\right)^{\gamma_2}, \end{cases} \Rightarrow \xi = |x|/\tau(t)^{1/p},$$

$$\gamma = \frac{p}{p-1}, \gamma_i = \frac{(p-1)[p-(m_i+1)]}{q}, \ i = 1,2,$$

$$q = (p-2)^2 - (m_1-1)(m_2-1).$$

Note that the functions $u_+(t,x),\ v_+(t,x)$ have the property

$u_+(t,x) \equiv 0,\ v_+(t,x) \equiv 0$ at $|x| \geq l(t), \quad v_+(t,x) \geq l(t) = a^{(p-1)/p}[\tau(t)]^{1/p}$.

Theorem 1. Let $m_i + k_i + l_i + p - 1 > 0, \quad \beta_i > \dfrac{m_{3-i}+k_i+l_i+p-1}{m_i+k_i+l_i+p-1}, \ i = 1,2,$

$\dfrac{\beta_1+1}{\beta_1\beta_2-1}a_1 \geq N/p, \quad \dfrac{\beta_2+1}{\beta_1\beta_2-1}a_2 \geq N/p$ and

$$\begin{cases} u_+(0,x) \ge u_0(x) \\ \\ v_+(0,x) \ge v_0(x) \end{cases}, \quad x \in R_+^2.$$

Then for sufficiently small $u_0(x)$, $v_0(x)$ the following estimates hold in Q,

$$\begin{cases} u(t,x) \ge A_1 u_+(t,x), \\ \\ v(t,x) \ge A_2 v_+(t,x) \end{cases} \tag{19}$$

to solve problem (1), (2), where $u_+(t,x)$, $v_+(t,x)$ are the functions defined above, $A_1 > 0, A_2 > 0$ are constants.

Theorem 2. Let

$m_i + k_i + l_i + p - 1 > 0$, $\beta_i > \dfrac{m_{3-i} + k_i + l_i + p - 1}{m_i + k_i + l_i + p - 1}$, $i = 1,2$, $a_1\left(\dfrac{\beta_1 + 1}{\beta_1\beta_2 - 1} + a^{q_2\beta_1 - q_1}\right) < \dfrac{N}{p}$,

$a_2\left(\dfrac{\beta_2 + 1}{\beta_1\beta_2 - 1} + a^{q_1\beta_2 - q_1}\right) < \dfrac{N}{p}$, $a_2\left(\dfrac{\beta_2 + 1}{\beta_1\beta_2 - 1} + a^{q_1\beta_2 - q_1}\right) < \dfrac{N}{p}$ and

$$\begin{cases} u_+(0,x) \ge u_0(x), \\ v_+(0,x) \ge v_0(x), \end{cases} \quad x \in R_+^2.$$

Then, for sufficiently small initial data $u_0(x)$, $v_0(x)$, the following estimates are valid for the weak solution at Q,

$$\begin{cases} u(t,x) \le A_3 u_+(t,x) \\ \\ v(t,x) \le A_4 v_+(t,x) \end{cases} \tag{20}$$

where $u_+(t,x)$, $v_+(t,x)$ - functions defined above, $A_3 > 0, A_4 > 0$ - constants.

From **Theorem 2**, we obtain the following value for the Fujita-type critical exponent:

$$\frac{\beta_i + 1}{\beta_i\beta_{3-i} - 1} = \frac{s}{p + (p + m_i - 3)s}, \quad i = 1,2$$

for system (1).

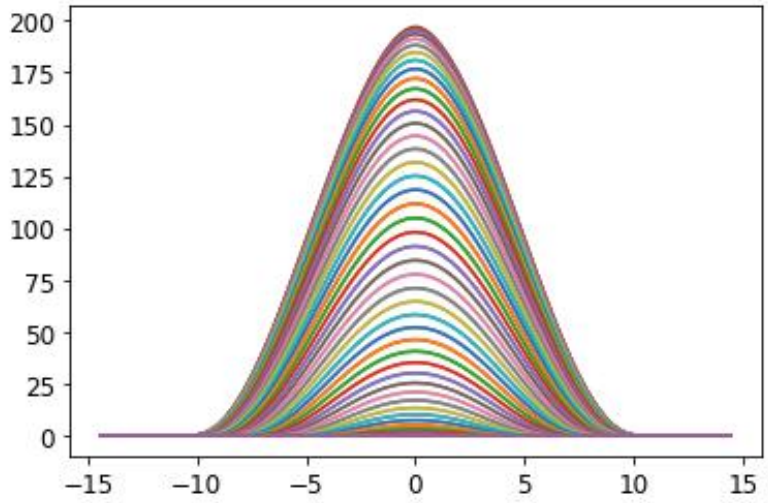
Results of numerical experiments and visualization. When solving the problem numerically, the equation is approximated on a grid using an implicit scheme of variable directions (for the multidimensional case) in combination with the balance method. Iterative processes were built on the basis of the Picard, Newton method as well as special method.

A computational algorithm has been developed. When developing software that illustrates the simulation process of solution behavior over time (visualization), the Anaconda 2024.06 environment was used with the inclusion of the Matplotlib.

The results of computational experiments show that all iterative methods are suitable for the constructed scheme. To achieve the same accuracy, the Newton method requires fewer iterations than the Picard method. In some cases, the number of iterations is almost twice, and the maximum number of iterations is 3-4 times less than in other methods. Since power-law nonlinearity is present in the right-hand side of equation (1), naturally the special method gives better results than the Picard method.

As a test example, we used solutions of equation (1) obtained by the methods of reference equations and nonlinear splitting [2]. Figures 1-4 show the calculation results for various values of parameters $m_1 = 2,\ p = 2.5,\ \beta_1 = 0.5,\ \varepsilon_1 = -1, k = 1.1,\ l = 1.2$ and time.

The results of a computational experiment for various values of the parameters included in the equation in the two-dimensional case are given below:



Conclusion. The results of the search for the solution of the given problem (1)-(2) show that it is necessary to analyze and find self-similar or approximately self-similar solutions for the study of nonlinear heat diffusion and nonlinear thermal conductivity processes. For this purpose, it is necessary to observe the qualitative properties of systems of equations, as well as cases of compact asymptotic support of non divergent solutions of systems of equations. New effects related to the problem were discovered in the search for solutions of self-similar models. From finite-difference schemes, it is clear that a nonlinear decomposition approach and a similar method can be developed in practice. In addition, the excellent results of creating iterative approaches using Newton's and Picard's methods in finding numerical solutions to systems of nonlinear standard equations lead to numerically computable solutions. Using the values of the specified parameters, it is possible to see the effect of the speed of heat propagation and the conditions of spatial localization.

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