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P-V CRITICALITY AND STABILITY ANALYSIS OF CHARGED BTZ BLACK HOLES

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Abstract. This work explores the thermodynamic properties of charged BTZ black holes in a (2+1)-dimensional extended phase space framework. By analyzing temperature-pressure relationships and heat capacity as functions of the event horizon radius, we identify critical points and examine Gibbs free energy to confirm stability. Our findings reveal significant similarities between charged BTZ black holes and other black hole solutions, enhancing our understanding of their thermodynamic behavior. This work contributes to theoretical physics and cosmology by uncovering complex interactions and providing insights into quantum gravity theories, while laying a foundation for future research in higher-dimensional systems.

Keywords: black holes, thermodynamic, event horizon, BTZ, charge, AdS.

Zaryadlangan BTZ qora tuynuklarning p-v kritikligi va barqarorlik tahlili

Annotatsiya. Ushbu ishda (2+1)-o‘lchovli kengaytirilgan fazoda zaryadlangan BTZ qora tuynuklarining termodinamik xususiyatlari o‘rganildi. Harorat va bosim o‘zaro bog‘liqligi hamda issiqlik sig‘imi hodisa gorizonti radiusiga bog‘liq holda tahlil qilinib, kritik nuqtalar aniqlanadi va Gibbs erkin energiyasi tahlil qilinib, barqarorlik tasdiqlanadi. Tadqiqot natijalari zaryadlangan BTZ qora tuynuklari va boshqa qora tuynuk yechimlari o‘rtasida sezilarli o‘xshashlik borligini ko‘rsatadi, bu esa ularning termodinamik xatti-harakatlarini yanada chuqurroq tushunishga yordam beradi. Ushbu ish nazariy fizika va kosmologiyaga o‘z hissasini qo‘shib, murakkab o‘zaro ta’sirlarni ochib beradi hamda kvant gravitatsiya nazariyalari bo‘yicha tushunchalar hosil qiladi, shuningdek, yuqori o‘lchovli tizimlarni tadqiq qilish uchun asos yaratadi.

Kalit so‘zlar: qora tuynuklar, termodinamika, hodisa gorizonti, BTZ, zaryad, AdS.

P-V критичность и анализ стабильности заряженных BTZ чёрных дыр

Аннотация. Данное исследование изучает термодинамические свойства заряженных BTZ-чёрных дыр в (2+1)-мерной расширенной фазовой пространственной структуре. Анализируя взаимосвязь температуры и давления, а также теплоёмкость в зависимости от радиуса горизонта событий, мы выявляем критические точки и исследуем свободную энергию Гиббса для подтверждения стабильности. Наши результаты показывают значительное сходство между заряженными BTZ-чёрными дырами и другими решениями чёрных дыр, углубляя понимание их термодинамического

поведения. Эта работа вносит вклад в теоретическую физику и космологию, раскрывая сложные взаимодействия и предоставляя ценные сведения о теориях квантовой гравитации, а также закладывает основу для дальнейших исследований в многомерных системах.

Ключевые слова: Черные дыры, термодинамика, горизонт событий, BTZ, заряд, АдС

Introduction. Black hole thermodynamics has become one of the most captivating fields in modern theoretical physics, connecting general relativity, quantum mechanics, and cosmology. Among various black hole solutions, the BTZ (Banados-Teitelboim-Zanelli) black hole in $(2+1)$-dimensional spacetime offers a unique opportunity to study these mysterious objects without the complexities of higher-dimensional systems[1, 2, 3]. This work focuses on charged BTZ black holes within the extended phase space framework, where the cosmological constant is treated as a thermodynamic pressure[4].

The foundation of black hole thermodynamics was laid in the 1970s by Bekenstein and Hawking, who demonstrated that black holes possess entropy proportional to their event horizon area and emit thermal radiation[5], now famously known as Hawking radiation. Since then, numerous studies have explored the thermodynamic behavior of black holes under different conditions. The BTZ black hole, characterized by its mass and angular momentum, provides a simplified model for understanding these phenomena. When electric charge is introduced, the system becomes even more complex and interesting, as electromagnetic fields interact with gravitational effects, leading to richer thermodynamic properties.

Lately, the lengthy stage space approach has acquired huge consideration in dark opening thermodynamics. This structure treats the cosmological consistency as a thermodynamic variable similar to pressure, presenting the idea of thermodynamic volume. For charged BTZ dark openings, this approach uncovers basic peculiarities like those seen in the Van der Waals fluid gas system[6]. Stage changes, basic focuses, and other intriguing ways of behaving arise when the interchange between temperature, strain, and charge is dissected. The presence of charge alters the thermodynamic way of behaving essentially, affecting how the framework answers changes in temperature and strain. As the charge expands, the framework's solidness is impacted, prompting potential stage transitions[7,8,9].

Several studies have contributed to our understanding of charged BTZ black holes. For example, [10] demonstrated Van der Waals-like behavior in AdS black holes, laying the groundwork for analyzing critical phenomena. [11] extended these ideas to Born-Infeld electrodynamics, revealing robust criticality in the thermodynamic behavior of black holes. Similarly, [12] explored the combined effects of Born-Infeld fields and quintessence dark energy, further enriching our understanding of black hole thermodynamics.

This study aims to contribute to the growing body of knowledge on charged BTZ black holes by providing a comprehensive analysis of their thermodynamic properties in the extended phase space framework. By investigating the interplay between temperature, pressure, and charge, we hope to enhance our understanding of black hole thermodynamics and its implications for fundamental physics. Furthermore, comparing the results with other black hole solutions could reveal universal patterns that deepen our comprehension of these enigmatic objects.

BTZ Black Holes

We consider the solution of charged black holes in $(2+1)$ dimensions with a negative cosmological constant in EinsteinMaxwell theory[13,14].

$$I = \int d^3x\sqrt{-g}\left(\frac{R-2\Lambda}{16\pi G} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right). \quad (1)$$

where R is the Ricci scalar, Λ is the cosmological constant, G is the gravitational constant, and $F_{\mu\nu}$ is the electromagnetic field strength tensor.

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\theta^2. \quad (2)$$

where the function $f(r)$ is

$$f(r) = -8GM + \frac{r^2}{l^2} - 8\pi GQ^2 \ln\left(\frac{r}{l}\right). \quad (3)$$

with M representing the mass of the black hole, Q its electric charge, and l the AdS radius related to the cosmological constant by $\Lambda = -1/l^2$.

P-V criticality

In the context of black hole thermodynamics within the extended phase space, the first law can be written as:

$$dM = TdS + VdP + \Phi dQ. \quad (4)$$

In this formulation, the black hole is treated as a thermodynamic system where the cosmological constant Λ is interpreted as a thermodynamic pressure P, and the conjugate volume V is associated with the black hole's size. This extension allows for a more comprehensive understanding of the thermodynamics of black holes, especially in the context of various modifications or generalizations of black hole theories.

$$P = -\Lambda, \quad (5)$$

$$V = \left(\frac{\partial M}{\partial P}\right). \quad (6)$$

With 5 , the mass can be reexpressed as

$$M = \frac{r^2}{8Gl^2} - \pi Q^2 \ln\frac{r}{l}. \quad (7)$$

As shown by Kubiznak and Mann [15], if one treats the cosmological constant as a thermodynamic pressure, a charged AdS BH displays analogical critical behavior with the Van der Waals liquid/gas system. This kind of $P - V$ criticality maintains in the AdS BHs with the presence of Born-Infeld field [16], power Maxwell source [17], power Yang-Mills field [18], quintessence dark energy [19. For the BTZ AdS BHs presented in the current paper, it is attractive for us to examine its thermodynamical phase transition by using $P - V$ criticality.

With Eqs. (5) and (6), the Hawking temperature can be reexpressed as

$$T = \frac{Pr_h}{2\pi} - \frac{2GQ^2}{r_h}. \quad (8)$$

From Eq. (8), one can easily derive the equation of state of the BH as

$$P = \frac{2\pi T}{r_h} - \frac{2\pi GQ^2}{r_h^2}. \quad (9)$$

The third term in the above expression for P reflects the effect of the BTZ. Note that the pressure P in extended phase space thermodynamics of asymptotically AdS BHs is entirely due to the variable cosmological constant, according to Eq. (9), thus the pressure of the matter fluid should appear in the equation of state of the BH. This is different from the case in horizon thermodynamics context, where P is identified with the total pressure of all matter in the spacetime, including the cosmological constant.

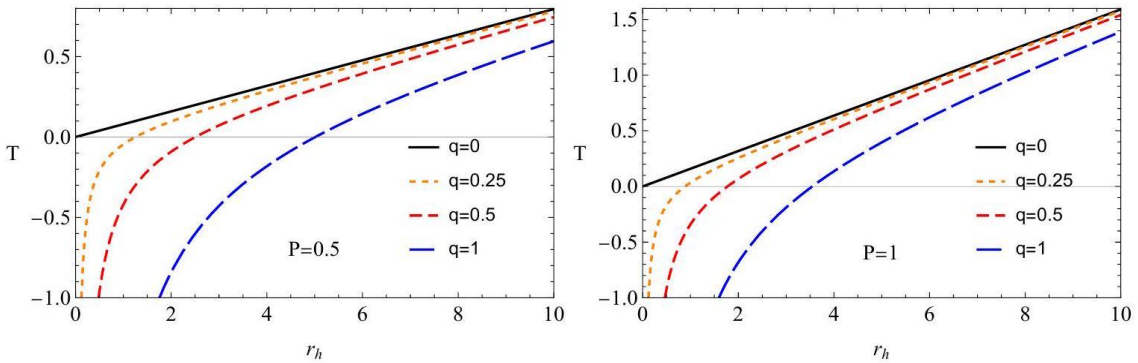

Figure 1. Temperature as a function of event horizon $\mathbf{r_h}$ for a charged BTZ black hole. Here, we consider $\mathbf{P = 0.5, 1}$ and different values of electric charge $\mathbf{Q}$. When $\mathbf{Q = 0}$, the temperature profile of BTZ black holes is recovered.

The graphs provide key insights into the thermodynamic and gravitational processes central to black hole physics. Fig. 1 how temperature varies with the event horizon radius r_h, reflecting the interplay between black hole entropy and Hawking radiation. The presence of electric charge reduces the temperature by extracting additional energy from the system via electromagnetic fields. As the horizon radius increases, this effect diminishes because the expansion of the horizon raises the internal temperature of the black hole.

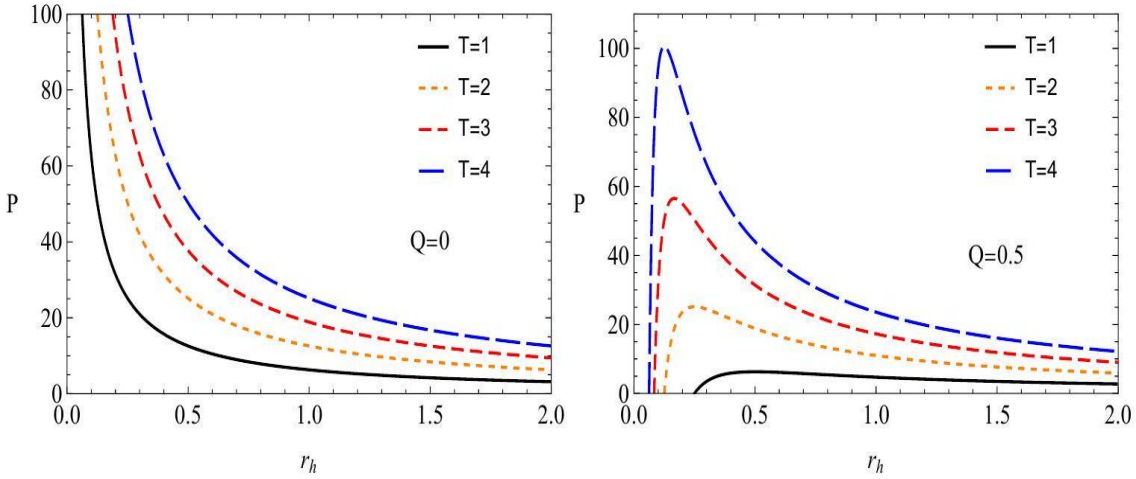

Figure 2. Pressure as a function of event horizon $\mathbf{r_h}$. Here we consider different values of $\mathbf{Q = 0.5}$ and temperature $\mathbf{T}$. When $\mathbf{Q = 0}$, we reconstruct the heat capacity profile of static BTZ black holes.

Fig. 2 shows the relationship between pressure (P) and the event horizon radius, highlighting the role of the cosmological constant in gravitational dynamics. Without charge, pressure decreases most rapidly, but the presence of charge further lowers the pressure due to the repulsive effects of electromagnetic fields. As temperature rises, the rate of pressure decreases slows, indicating the establishment of thermal equilibrium within the system. Therefore, there is a region with nonphysical meaning where the thermodynamics description breaks down. As in the nonlinear Coulomb-like case, to understand this result and study the thermodynamics stability of this solution, we compute heat capacity and move toward understanding possible critical behavior, followed by a comparison of our results with charged BTZ black holes

$$C = T\left(\frac{\partial S}{\partial T}\right) = \frac{2\pi r_h^2\left(Pr_h^2 - 4\pi Q^2\right)}{Pr_h^2 + 4\pi Q^2}. \qquad (10)$$

As T it increases, the pressure variation concerning r_h is shown. These changes indicate a decrease in pressure and a gradual flattening. When $Q = 0.5$ the pressure variation changes slightly more strongly and has a larger effect than expected compared to $Q = 0$. The left graph shows that the pressure decreases with increasing values T and remains flat for a black hole with no charge. In the right graph, the presence of the charge lowers the pressure a little faster, and the changes in T values are more pronounced.

Fig. 3 shows the heat capacity (C) as a function of the event horizon radius, revealing critical aspects of black hole stability and phase transitions. In the absence of charge, heat capacity increases steadily with radius, maintaining system stability. However, the presence of charge introduces regions where phase transitions may occur, as evidenced by divergences in heat capacity. Higher pressure accelerates these phase transitions by enhancing the gravitational effects of the cosmological constant.

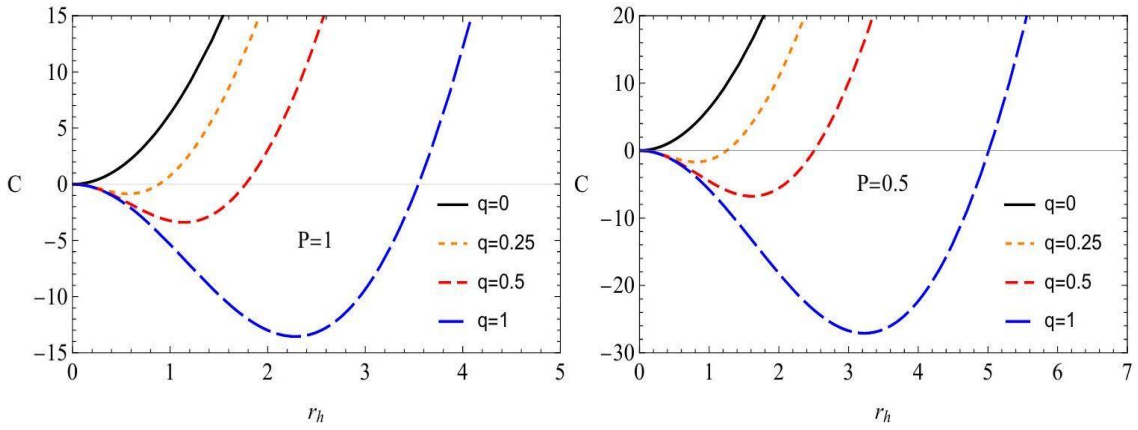

Figure 3. Heat capacity as a function of the event horizon $\mathbf{r_h}$. Here, we consider $\mathbf{P = 0.5, 1}$ different values of electric charge $\mathbf{Q}$. When $\mathbf{Q = 0}$ we recover the heat capacity profile of static BTZ black holes.

Thus we confirm that there are no major differences in the standard thermodynamics descriptions between charged BTZ black holes and Coulomb-like $2+1$ dimensional black holes: both have thermodynamic stability.

The critical point can be derived through the following conditions

$$\left.\frac{\partial P}{\partial r_h}\right|_{T=T_c} = 0, \tag{11}$$

$$\left.\frac{\partial^2 P}{\partial r_h^2}\right|_{T=T_c} = 0. \tag{12}$$

Implementing Eqs.11, Eqs. 12 and exploring the characteristics of the critical phase transition can lead to the set of higherdimensional critical triplets (T_c, P_c, r_c) in the guise of

$$T_c = \frac{2GQ^2}{r_h}, \tag{13}$$

$$P_c = \frac{2\pi GQ^2}{r^2}. \tag{14}$$

To understand this phase transition more deeply, one can analyze the behavior of Gibbs free energy. In the extended phase space, the mass is interpreted as enthalpy. So the Gibbs free energy can be derived as

$$G = \pi Q^2 - \frac{r_h^2}{8Gl^2} - \pi Q^2 \ln\frac{r_h}{l}. \tag{15}$$

We now analyze our solution's global stability by investigating Gibbs free energy. This thermodynamic potential is calculated from the Euclidean action and an appropriate cut-off term. A comprehensive stability analysis can be done using the Gibbs free energy sign. In the extended phase space, $G = H - TS = M - TS$ gives the Gibbs free energy. Practically speaking, it should be noted that any discontinuous behavior in the first- or second-order derivatives of the Gibbs energy leads to a firstor second-order phase transition in the system.

Conclusion. This study has comprehensively explored the thermodynamic properties of charged BTZ black holes within the extended phase space framework, treating the cosmological constant as thermodynamic pressure. Our analysis revealed that electric charge significantly affects the temperature, pressure, and heat capacity of these black holes. Specifically, the presence of charge reduces the temperature at smaller event horizon radii, with this effect diminishing as the horizon expands. Pressure dynamics showed that charge lowers the pressure exerted on the black hole, particularly at smaller radii, while higher temperatures slow down the pressure decrease, indicating a gradual approach toward equilibrium. Heat capacity analysis identified critical points where phase transitions may occur, highlighting regions of instability near these points. These findings confirm the stability of charged BTZ black holes under certain conditions but also reveal complex interactions

between charge, pressure, and temperature. Despite these insights, further research is needed to investigate higher-order corrections to the Einstein-Maxwell theory, explore holographic implications, and extend the analysis to rotating charged BTZ black holes. Overall, this work contributes to the understanding of black hole thermodynamics and lays a foundation for future studies in fundamental physics.

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