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SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ ШАРОФА РАШИДОВА

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ILMIY AXBOROTNOMA

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Obuna indeksi – yakka tartibdagi obunachilar uchun **- 5583,**
tashkilot, korxonalar uchun **- 5584**

MUNDARIJA

Икромов И.А, Баракаев А.М. *Об ограниченности максимальных операторов для выпуклых функций, не имеющих конечный линейный тип*…….....	3-10
Safarov A.N., Muzafarov A.M., To‘xtayev U.O‘., Tog‘ayev B.S., Salimov Sh.G‘., Esanov I.T. *Tabiiy suvlar tarkibidagi radionuklidlar miqdorini radiokimyoviy radiometriya usullari yordamida aniqlash metodikasi*……..	11-20
Ruziyev J.E., Abduraxmanov I.E., Suvonqulov Sh.B. *Analitik obyektlardagi bariy ionlarini ionometrik aniqlash*……..	21-25
O‘rozov S., Aslanov Q., Qo‘ldoshev O. *Oltingugurt qoplamali fosforli o‘g‘itlar olish*……..	26-30
Mamatov Sh., Omonov Sh. *Differensial tenglamalar misolida fanlararo bog‘lanish*……..	31-35
Ахмеджанов Ф.Р., Тугалов Ф.Б., Курбанов Ж.О. *$NaCl_xBr_{1-x}$ кристалларининг акустик хоссалари*……	36-40
Inxonova A., Eshtursunov D., Botirov S., Murtazaqulov M., Bekchanov D. *Oqava suvlarni biologik tozalashda pvc&pei asosida olingan mis polimer komlpleksining qo’llanilishi*……	41-48
Qurbonov P.R. *O‘zbekiston respublikasi urbanistik tarkibi rivojlanishining geografik jihatlari*.....	49-55
Ikromov I.A., Jo‘raqulov A.J. *Regulyarlashtirilgan o‘lchovning bahosi haqida*……	56-63
Байжуманов А.А., Бердимуродов М.А. *Метод решения систем нелинейных булевых уравнений на основе минимизации дизъюнкций сложных конъюнкций*……	64-70
Abdullayev J.U. *Uglevodorodlar asosida spirtlar sintezi*……	71-75
Boyqobilov D.B., Jumag‘ulov Sh.H., Ro‘zimuradov O.N. *Tartiblangan titan dioksid nanotrubkalarining shakllanishi*……	76-80
Ахмеджанов Ф.Р., Мустафаев Т.Ш., Эшниязов О.И., Нуждов Г.С., Авдиевич В.Н. *Лантан трифторид кристалларида ионларнинг харакатига эластик деформациянинг таъсири*……	81-85
Хайдаров И.К. *Моделирование переноса растворенного вещества в неоднородном пороупругом глинистом сланце*……	86-91
Urolov Sh.Z., Shaymardanov Z.Sh., Jalolov R.R., Rustamova B.N., Arziqulov E.U. *Rux oksidi nanostrukturalarining morfologik va kristallografik xossalarining sintez sharoitiga bog‘liqligi*……	92-99
Zufarov A.M., Sagdeyev D.O., Galyametdinov Yu.G., Muxamadiyev N.K. *Turli xil anion stabilizatorlar bilan stabillashgan* CdS *sintezi va uning fotokatalitik xossalari*……	100-108

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REGULYARLASHTIRILGAN O‘LCHOVNING BAHOSI HAQIDA

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Annotatsiya. Bu ishda regulyarlashtirilgan o‘lchov zichlik funksiyasi uchun uch o‘lchamli Evklid fazosida baholar olinadi. Zichlik funksiyasi, kompleks qiymatli funksiya bo‘lib, Schwartz fazosiga qarashlidir. Bu kabi baholar sirt o‘lchovining cho‘zishi bilan bog‘langan maksimal operatorlarning Lebeg fazolarida chegaralanganligini isbotlashda qo‘llaniladi.

Kalit so‘zlar: Furye almashtirishi, kesuvchi funksiyalar, o‘lchovning regulyarlashgani, asimptotik baho, statsionar faza usuli.

Об оценке регуляризованной меры

Аннотация. В данной работе получены оценки для функции плотности регуляризованной меры в трехмерном Евклидовом пространстве. Функция плотности является комплексно-значной функцией, принадлежашей в пространству Шварца. Такие оценки используются при доказательстве ограниченности максимальных операторов, связанных с растяжением поверхностной меры, в пространствах Лебега.

Ключевые слова: Преобразование Фурье, секущая функции, регуляризованная мера, асимптотическая оценка, метод стационарной фазы.

On estimates for the regularized measure

Abstract. In this article, we obtain estimate for the density function of the regularized measure in three-dimensional Euclidean space. The complex-valued density function belongs to the Schwartz space. Such estimates are used to prove the boundedness of the maximal operators associated with dilation of surface-carried measure in Lebesgue spaces.

Keywords: Fourier transform, cut-off functions, regularized measure, asymptotic estimate, stationary phase method.

Masalaning qo‘yilishi va asosiy teorema. Bu ishda regulyarlashtirilgan o‘lchov zichlik funksiyasi uchun uch o‘lchamli Evklid fazosida baholar olinadi. Zichlik funksiyasi, kompleks qiymatli funksiya bo‘lib, Schwartz fazosiga qarashlidir. Bu kabi baholar sirt o‘lchovining cho‘zishi bilan bog‘langan maksimal operatorlarning Lebeg fazolarida chegaralanganligini isbotlashda qo‘llaniladi.

Dastlab masalaning qo‘yilishi bilan shug‘ullanamiz.

Bizga quyidagi sirt berilgan bo‘lsin.

$$S \coloneqq \{(x_1, x_2, x_3) \in \mathbb{R}^3, x_3 = 1 + \Phi(x_1, x_2)\}$$

bu yerda, $(x_1, x_2) \in U \subset \mathbb{R}^2$, U $(0,0)$ nuqtaning yetarli kichik atrofi. Bu atrofda $\Phi(x_1, x_2)$ funksiya yetarlicha silliq bo‘lib, $\Phi(0,0) = 0, \nabla\Phi(0,0) = 0$ shartlarni qanoatlantiradi.

Faraz qilaylik, yuqoridagi sirtda aniqlangan $\psi(x) \in C_0^\infty(S)$ funksiya berilgan bo‘lib, u $(0,0,1)$ nuqtaning yetarli kichik atrofida mujassamlashgan bo‘lsin.

Shu sirtda $\mu \coloneqq \psi(x)dS$ zaryad (ishora almashinuvchi o‘lchov)ning Furye almashtirishini [3, 41-b] qaraymiz, ya’ni

$$\widehat{\mu(\xi)} = \int_S e^{-i(\xi,x)}\, \psi(x)dS(x).$$

Osongina ko‘rish mumkinki $\widehat{\mu(\xi)}$ $\mathbb{C}^3$ da analitik, ya’ni butun funksiya bo‘ladi.

Yuqoridagi Furye almashtirishini aniqroq quyidagi ikki karrali integral shaklida yozamiz:

$$\widehat{\mu(\xi)} = \int_{\mathbb{R}^2} e^{-i(\xi_1 x_1 + \xi_2 x_2 + \xi_3(1+\Phi(x_1,x_2)))} a(x_1, x_2)dx_1 dx_2. \qquad 1)$$

Bunda, yozuvni qisqartirish maqsadida quyidagi belgilashni kiritdik:

$$a(x_1, x_2) = \psi(x_1, x_2, 1 + \Phi(x_1, x_2))\sqrt{1 + |\nabla\Phi(x_1, x_2)|^2}\ .$$

Endi, $\widehat{\mu^{\lambda}(\xi)}$ deb quyidagini olamiz:

$$\widehat{\mu^{\lambda}(\xi)} := \chi_1\left(\frac{\xi_3}{\lambda}\right)\chi_0\left(\frac{\xi_1}{\lambda},\frac{\xi_2}{\lambda}\right)\widehat{\mu(\xi)} \tag{2}$$

bu yerda, $\lambda \gg 1$ parametr bo‘lib, $\chi_1\left(\frac{\xi_3}{\lambda}\right)$ va $\chi_0\left(\frac{\xi_1}{\lambda},\frac{\xi_2}{\lambda}\right)$ funksiyalar quyidagi shartlarni qanoatlantiruvchi no-manfiy cheksiz silliq funksiyalar:

$$\chi_0\left(\frac{\xi_1}{\lambda},\frac{\xi_2}{\lambda}\right) = \begin{cases} 1, & \left|\left(\frac{\xi_1}{\lambda},\frac{\xi_2}{\lambda}\right)\right| \le 1 \\ 0, & \left|\left(\frac{\xi_1}{\lambda},\frac{\xi_2}{\lambda}\right)\right| > 2 \end{cases}$$

$$\chi_1\left(\frac{\xi_3}{\lambda}\right) = \chi_0\left(\frac{\xi_3}{\lambda}\right) - \chi_0\left(\frac{2\xi_3}{\lambda}\right).$$

Bu kabi funksiyalarga kesuvchi (“cut-off functions”) funksiyalar deb yuritiladi.

$\widehat{\mu^{\lambda}(\xi)}$ kompakt tashuvchili $C_0^{\infty}(\mathbb{R}^3)$ funksiya bo‘lgani bois uning teskari Furye almshtirishi- μ^{λ} Schwartz funksiyasi ekanligi ravshan. Unga μ o‘lchovning regulyarlashganining zichlik funksiyasi deb aytaymiz [1]. Biz bu funksiyaning $\lambda \gg 1$ bo‘lgandagi xarakteri bilan shug‘ullanamiz.

Biz $\widehat{\mu^{\lambda}(\xi)}$ uchun teskari Furye almashtirishini olamiz.

$$\mu^{\lambda}(y) = \frac{1}{(2\pi)^3}\int_{\mathbb{R}^3} e^{i(\xi,y)}\widehat{\mu^{\lambda}(\xi)}\, d\xi. \tag{3}$$

Endi, ishning asosiy natijasi bo‘lgan teoremani keltiramiz:

Asosiy teorema: *(0,0,1) nuqtaning shunday V atrofi topiladiki, istalgan* $\psi \in C_0^{\infty}(V)$ va $|\lambda| > 1$ *uchun quyidagi*

$$\left|\mu^{\lambda}(y)\right| \le C|\lambda|,$$

baho o‘rinli bo‘ladi. Bunda C y va λ larga bog‘liq bo‘lmagan o‘zgarmas son.

Bu kabi baholar sirt o‘lchovi bilan bog‘langan maksimal operatorlarning Lebeg fazolarida chegaralanganligini isborlashda foydalaniladi ([1] ga qarang).

2. Asosiy teoremaning isboti. (3) tenglikning o‘ng tomonini (2) va (1) munosabatlardan foydalanib quyidagicha yozamiz.

$$\mu^{\lambda}(y) = \frac{1}{(2\pi)^3}\int_{\mathbb{R}^5} \chi_1\left(\frac{\xi_3}{\lambda}\right)\chi_0\left(\frac{\xi_1}{\lambda},\frac{\xi_2}{\lambda}\right)a(x_1,x_2)\times$$
$$\times\, e^{i(\xi_1(y_1-x_1)+\xi_2(y_2-x_2)+\xi_3(y_3-(1+\Phi(x_1,x_2)))}dx_1dx_2d\xi_1d\xi_2d\xi_3. \tag{4}$$

Yuqoridagi integralda quyidagi o‘zgaruvchilarni almashtirishni olamiz.

$$\begin{cases} \xi_1 = \lambda s_1 s_3 \\ \xi_2 = \lambda s_2 s_3 \\ \xi_3 = \lambda s_3 \end{cases} \tag{5}$$

$$\frac{D(\xi_1,\xi_2,\xi_3)}{D(s_1,s_2,s_3)} = \lambda^3 s_3^2.$$

(5) almashtirish orqali (4) integral quyidagi ko‘rinishga keladi:

$$\mu^{\lambda}(y) = \frac{\lambda^3}{(2\pi)^3}\int_{\mathbb{R}^5} \chi_1(s_3)\chi_0(s_1s_3,s_2s_3)a(x_1,x_2)\, s_3^2\times$$
$$\times\, e^{i\lambda s_3(s_1(y_1-x_1)+s_2(y_2-x_2)+(y_3-(1+\Phi(x_1,x_2)))}dx_1dx_2ds_1ds_2ds_3. \tag{6}$$

Faraz qilaylik, $|y_1| + |y_2| + |y_3 - 1| > 3\delta$ bunda δ musbat son. Umumiylikka zarar yetkazmagan holda $|y_i| > \delta$ va $|x_i| < \frac{\delta}{4}$ deb olsak, u holda quyidagi

$$\left|\frac{\partial(s_1(y_1-x_1)+s_2(y_2-x_2)+(y_3-(1+\Phi(x_1,x_2)))}{\partial s_i}\right| > \frac{\delta}{2} \tag{7}$$

tengsizlikka ega bo‘lamiz, bunda $i = 1,2$.

Xuddi shunday $|s_i| > \delta$ *va* $|x_i| < \frac{\delta}{4}$ (ya'ni $a(x_1, x_2)$ funksiyaning tashuvchisi "support"i $\left(-\frac{\delta}{4}, \frac{\delta}{4}\right) \times \left(-\frac{\delta}{4}, \frac{\delta}{4}\right)$ kvadratda yotadi deb faraz qilamiz, tabiiyki δ musbat soni topilib, u $\nabla\Phi(x_1, x_2)$ vektor funksiyaga bog'liq i=1,2) bo'lganda

$$\left|\frac{\partial(s_1(y_1 - x_1) + s_2(y_2 - x_2) + (y_3 - (1 + \Phi(x_1, x_2)))}{\partial x_i}\right| > \frac{\delta}{2} \tag{8}$$

tengsizlikka ega bo'lamiz.

Nihoyat, $|y_3 - 1| > \delta$ va $|\Phi(x_1, x_2)| < \frac{\delta}{4}$ deb olganimizda (7) tengsizlikka o'xshash quyidagi,

$$|s_1(y_1 - x_1) + s_2(y_2 - x_2) + (y_3 - (1 + \Phi(x_1, x_2))| > \frac{\delta}{2} \tag{9}$$

quyi bahoga ega bo'lamiz. Yuqoridagi shartlar asosida (6) integralni baholashda s_j $(j = 1, 2, 3)$ va x_i $(i = 1, 2)$ o'zgaruvchilar bo'yicha bo'laklab integrallash formulasini qo'llaymiz. Yozuvni qisqartirish maqsadida quyidagi almashtirishni olamiz:

$$\begin{cases} x_1 = u_1 \\ x_2 = u_2 \\ s_1 = u_3 \\ s_2 = u_4 \\ s_3 = u_5 \end{cases} \tag{10}$$

$$\frac{D(x_1, x_2, s_1, s_2, s_3)}{D(u_1, u_2, u_3, u_4, u_5)} = 1$$

(10) almashtirish orqali (6) integral quyidagi ko'rinishga keladi:

$$\mu^{\lambda}(y) = \frac{\lambda^3}{(2\pi)^3} \int\limits_{\mathbb{R}^5} \alpha(u_1, u_2, u_3, u_4, u_5)\, e^{i\lambda\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)} du_1 du_2 du_3 du_4 du_5, \tag{11}$$

bunda:

$$\alpha(u_1, u_2, u_3, u_4, u_5) = \chi_1(u_5)\chi_0(u_3u_5, u_4u_5)a(u_1, u_2)u_5^2,$$

$$\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3) := u_5(u_3(y_1 - u_1) + u_4(y_2 - u_2) + (y_3 - (1 + \Phi(u_1, u_2)))$$

Endi, yuqoridagi integral ostidagi funksiya uchun [2, 116-b] dagi 2.1-lemmadan foydalanib bo'laklab integrallash formulasini qo'llaymiz:

$$\int\limits_{\mathbb{R}^5} \alpha(u_1, u_2, u_3, u_4, u_5)\, e^{i\lambda\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)} du_1 du_2 du_3 du_4 du_5 =$$

$$= \frac{1}{i\lambda} \int\limits_{\mathbb{R}^5} \alpha(u_1, u_2, u_3, u_4, u_5) \times$$

$$\times \frac{\sum_{j=1}^{5} \frac{\partial\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)}{\partial u_j} \frac{\partial e^{i\lambda\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)}}{\partial u_j}}{\sum_{j=1}^{5} \left(\frac{\partial\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)}{\partial u_j}\right)^2} du_1 du_2 du_3 du_4 du_5 =$$

$$= \frac{1}{i\lambda} \int\limits_{\mathbb{R}^5} \left(\sum_{j=1}^{5} \frac{\partial}{\partial u_j} \left(\frac{\frac{\partial\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)}{\partial u_j}}{\sum_{j=1}^{5} \left(\frac{\partial\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)}{\partial u_j}\right)^2} \alpha(u_1, u_2, u_3, u_4, u_5) \right) \right) \times$$

$$\times e^{i\lambda\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)}\, du_1 du_2 du_3 du_4 du_5 \tag{12}$$

(7), (8) va (9) shartlarni hisobga olsak yuqorida integral ostidagi xususiy hosilalar chegaralanganligiga kelamiz bundan esa quyidagi bahoga ega bo'lamiz.

$$\left| \int\limits_{\mathbb{R}^5} \alpha(u_1, u_2, u_3, u_4, u_5)\, e^{i\lambda\gamma(u_1, u_2, u_3, u_4, u_5, y_1, y_2, y_3)} du_1 du_2 du_3 du_4 du_5 \right| \leq$$

$$\leq C_1\lambda^{-1} \|\alpha(u_1, u_2, u_3, u_4, u_5)\|_{C^1(\mathbb{R}^5)}$$

13)

(12) tenglikdagi oxirgi integrallar uchun yana bo‘laklab integrallash formulasini qo‘llab, (13) ko‘rinishdagiga o‘xshash baho olamiz. Va hakazo (faza funksiyasi $N+4$ marta uzluksiz differensiallanuvchi va amplituda funksiyasi $N+3$ marta uzluksiz differensiallanuvchi va tashuvchisi $\left(-\frac{\delta}{4},\ \frac{\delta}{4}\right)\times\left(-\frac{\delta}{4},\ \frac{\delta}{4}\right)$ kvadratda yotgan holda) ketma-ket $(N+3)$ marta bo‘laklab integrallash formulasini qo‘llab, $|\lambda|\to+\infty$ da quyidagi bahoga ega bo‘lamiz:

$$\left|\mu^{\lambda}(y)\right|<\frac{C(N)}{|\lambda|^{N}}.$$

Demak, bu holda yuqoridagi teoremada keltirilgan bahodan ko‘ra yaxshiroq bahoga ega bo‘lamiz. Shuning uchun biz bundan keyin biror chekli yetarli kichik δ musbat soni uchun

$|y_1|+|y_2|<\delta$ va $|y_3-1|<\delta$ va $|s_1|+|s_2|<\delta$ deb faraz qilamiz.

Yuqoridagi karrali integralning asimptotik xarakterini hisoblashda dastlab (x_1,x_2,s_1,s_2) o‘zgaruvchilar bo‘yicha olingan to‘rt karrali integralni qaraymiz va bunda “statsionar faza” usulidan foydalanamiz.

$$\mu^{\lambda}(y)=\frac{\lambda^3}{(2\pi)^3}\int\limits_{\mathbb{R}^5}\chi_1(s_3)\chi_0(s_1s_3,s_2s_3)a(x_1,x_2)\,s_3^2\times$$
$$\times\,e^{i\lambda s_3(s_1(y_1-x_1)+s_2(y_2-x_2)+(y_3-(1+\Phi(x_1,x_2)))}dx_1dx_2ds_1ds_2ds_3=$$
$$=\frac{\lambda^3}{(2\pi)^3}\int\limits_{\mathbb{R}}\chi_1(s_3)\,s_3^2ds_3\int\limits_{\mathbb{R}^4}\chi_0(s_1s_3,s_2s_3)\times$$
$$\times\,a(x_1,x_2)e^{i\lambda s_3(s_1(y_1-x_1)+s_2(y_2-x_2)+(y_3-(1+\Phi(x_1,x_2)))}dx_1dx_2ds_1ds_2.$$

14)

Yuqoridagi ichki to‘rt karrali integralning $|\lambda|\to+\infty$ dagi asimptotikasini hisoblaymiz va hisoblashda [2, 117-b] 2.1- teoremadan foydalanamiz.

$$J(\lambda s_3,y)=\int\limits_{\mathbb{R}^4}\chi_0(s_1s_3,s_2s_3)\,a(x_1,x_2)\times$$
$$\times\,e^{i\lambda s_3(s_1(y_1-x_1)+s_2(y_2-x_2)+(y_3-(1+\Phi(x_1,x_2)))}dx_1dx_2ds_1ds_2.$$

15)

(15) integralda faza funksiyasi sifatida quyidagini olishimiz mumkin:

$F(x_1,x_2,s_1,s_2,y_1,y_2,y_3)=s_1(y_1-x_1)+s_2(y_2-x_2)+(y_3-(1+\Phi(x_1,x_2))$ (16)

Shuningdek, amplituda funksiyasi sifatida esa,

$$A(x_1,x_2,s_1,s_2,s_3)=\chi_0(s_1s_3,s_2s_3)a(x_1,x_2)$$

17)

funksiyani olishimiz mumkin.

Endi, faza funksiyasining kritik nuqtalarini topamiz.

$$\begin{cases}\frac{\partial F}{\partial x_1}=-s_1-\frac{\partial\Phi}{\partial x_1}=0\\ \frac{\partial F}{\partial x_2}=-s_2-\frac{\partial\Phi}{\partial x_2}=0\\ \frac{\partial F}{\partial s_1}=y_1-x_1=0\\ \frac{\partial F}{\partial s_2}=y_2-x_2=0\end{cases}$$

U holda quyidagiga ega bo‘lamiz:

$$\begin{cases}x_1=y_1\\ x_2=y_2\\ s_1=-\frac{\partial\Phi(y_1,y_2)}{x_1}\\ s_2=-\frac{\partial\Phi(y_1,y_2)}{x_2}\end{cases}$$

Ravshanki, yuqoridagi tenglamaning yechimi

$$(x_1^c,x_2^c,s_1^c,s_2^c)=(y_1,y_2,-\frac{\partial\Phi(y_1,y_2)}{x_1},-\frac{\partial\Phi(y_1,y_2)}{x_2})$$

bo‘ladi.

Endi, faza funksiyasining Hessianini hisoblaymiz, ya’ni:

$$\det(Hess(F)) = \begin{Vmatrix} \frac{\partial^2 F}{\partial x_1^2} & \frac{\partial^2 F}{\partial x_1 \partial x_2} & \frac{\partial^2 F}{\partial x_1 \partial s_1} & \frac{\partial^2 F}{\partial x_1 \partial s_2} \\ \frac{\partial^2 F}{\partial x_2 \partial x_1} & \frac{\partial^2 F}{\partial x_2^2} & \frac{\partial^2 F}{\partial x_2 \partial s_1} & \frac{\partial^2 F}{\partial x_2 \partial s_2} \\ \frac{\partial^2 F}{\partial s_1 \partial x_1} & \frac{\partial^2 F}{\partial s_1 \partial x_2} & \frac{\partial^2 F}{\partial s_1^2} & \frac{\partial^2 F}{\partial s_1 s_2} \\ \frac{\partial^2 F}{\partial s_2 \partial x_1} & \frac{\partial^2 F}{\partial s_2 \partial x_2} & \frac{\partial^2 F}{\partial s_2 \partial s_1} & \frac{\partial^2 F}{\partial s_2^2} \end{Vmatrix} =$$

$$= \begin{Vmatrix} -\frac{\partial^2 \Phi}{\partial x_1^2} & -\frac{\partial^2 \Phi}{\partial x_1 \partial x_2} & -1 & 0 \\ -\frac{\partial^2 \Phi}{\partial x_2 \partial x_1} & -\frac{\partial^2 \Phi}{\partial x_2^2} & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{Vmatrix} = 1$$

Yozuvni qisqartirish maqasadida quyidagi belgilashlarni kiritamiz:

$$-\frac{\partial^2 \Phi}{\partial x_1^2} = a_{11}, -\frac{\partial^2 \Phi}{\partial x_1 \partial x_2} = a_{12} = -\frac{\partial^2 \Phi}{\partial x_2 \partial x_1}, -\frac{\partial^2 \Phi}{\partial x_2^2} = a_{22}$$

$Hess(F)$ matritsaning xos qiymatlarini topamiz.

$$\det(Hess(F) - IE) = \begin{Vmatrix} a_{11} - E & a_{12} & -1 & 0 \\ a_{12} & a_{22} - E & 0 & -1 \\ -1 & 0 & -E & 0 \\ 0 & -1 & 0 & -E \end{Vmatrix}$$

$\det(Hess(F) - IE) = 0$ bu tenglamani quyidagi algebraik tenglama ko‘rinishida yozamiz:

$$E^4 - (a_{11} + a_{22})E^3 + (a_{11}a_{22} - a_{12}^2 - 2)E^2 + (a_{11} + a_{22})E + 1 = 0.$$

Yuqoridagi algebraik tenglamani yechimlarini “WolframAlpha” [4] sayitidan foydalangan holda olamiz:

$$E_1 = -\frac{1}{2}\sqrt{\frac{-(a_{11}+a_{22})\sqrt{(a_{11}-a_{22})^2+4a_{12}^2}+a_{11}^2+a_{22}^2+2a_{12}^2+8}{2}} - \\ -\frac{1}{4}\sqrt{(a_{11}-a_{22})^2+4a_{12}^2}+\frac{(a_{11}+a_{22})}{4},$$

$$E_2 = -\frac{1}{2}\sqrt{\frac{-(a_{11}+a_{22})\sqrt{(a_{11}-a_{22})^2+4a_{12}^2}+a_{11}^2+a_{22}^2+2a_{12}^2+8}{2}} + \\ +\frac{1}{4}\sqrt{(a_{11}-a_{22})^2+4a_{12}^2}+\frac{(a_{11}+a_{22})}{4},$$

$$E_3 = \frac{1}{2}\sqrt{\frac{-(a_{11}+a_{22})\sqrt{(a_{11}-a_{22})^2+4a_{12}^2}+a_{11}^2+a_{22}^2+2a_{12}^2+8}{2}} - \\ -\frac{1}{4}\sqrt{(a_{11}-a_{22})^2+4a_{12}^2}+\frac{(a_{11}+a_{22})}{4},$$

$$E_4 = \frac{1}{2}\sqrt{\frac{-(a_{11}+a_{22})\sqrt{(a_{11}-a_{22})^2+4a_{12}^2}+a_{11}^2+a_{22}^2+2a_{12}^2+8}{2}} + \\ +\frac{1}{4}\sqrt{(a_{11}-a_{22})^2+4a_{12}^2}+\frac{(a_{11}+a_{22})}{4}.$$

$Hess(F)$ matritsaning musbat va manfiy xos qiymatlarining sonini aniqlash maqsadida quyida bir lemma va uning natijasini keltiramiz.

Lemma: Faraz qilaylik, A simmetrik matritsaning determinanti noldan farqli bo‘lsin. Agar shunday $x \neq 0$ vektor uchun $(Ax, x) = 0$ bo‘lsa, u holda A matritsaning musbat va manfiy xos qiymatlari mavjud bo‘ladi.

Isbot: Chiziqli algebra kurisdan ma’lumki, simmtrik matritsaning determenanti noldan farqli bo’lsa, u holda uning barcha xos qiymatlari nolmas haqiqiydir**.** Faraz qilaylik, A simmetrik

matritsaning barcha xos qiymatlari musbat yoki manfiy bo‘lsin. U holda, ixtiyoriy $x \neq 0$ vektorlar uchun mos ravishda $(Ax, x) > 0$ yoki $(Ax, x) > 0$ tengsizliklar bajariladi. Bu esa lemma shartiga ziddir. Demak, biror $x \neq 0$ vektor uchun $(Ax, x) = 0$ bo‘lsa, u holda A matritsaning musbat va manfiy xos qiymatlari mavjud bo‘lar ekan. Lemma isbotlandi.

Natija: Yuqorida keltirilgan $Hess(F)$ matritsa ikkita musbat va ikkita manfiy xos qiymatga ega.

Isbot: Ma’lumki, $Hess(F)$ simmetrik matritsadir. Shuningdek, $\det\big(Hess(F)\big) = 1$. Agar biz $x = (0,0,1,1) \neq 0$ vektorni olsak, u vektor uchun $(Hess(F)(x), x) = 0$ tenglikka ega bo‘lamiz. Yuqoridagi lemmaga ko‘ra $Hess(F)$ simmetrik matritsaning hech bo‘lmaganda bittadan musbat va manfiy xos qiymatlari mavjud bo‘ladi. Chiziqli algebra kursidan ma’lumki, simmetrik matritsaning barcha xos qiymatlari haqiqiy bo‘lib, ularning ko‘paytmasi matritsaning determenantiga teng. Budan kelib chiqadiki,

$$\prod_{j=1}^{4} E_j = \det\big(Hess(F)\big) = 1. \tag{18}$$

Umumiylikka zarar yetkazmagan holda, $E_1 < 0$ va $E_4 > 0$ deb faraz qilamiz. U holda, (18) tenglikka ko‘ra, $E_2 E_3 < 0$ tengsizlikka ega bo‘lamiz. Bundan esa, E_2 va E_3 xos qiymatlar har xil ishorali ekanligi kelib chiqadi. Demak, $Hess(F)$ matritsa ikkita musbat xos qiymatga va ikkita manfiy xos qiymatga ega. Natija isbotlandi.

Yuqorida keltirilgan natijaga asosan $Hess(F)$ matritsaning signaturasi $sgn\big(Hess(F)\big) = 0$ ekan.

Yuqoridagilardan foydalanib (15) integralning $\lambda \gg 1$ bo‘yicha asimptotikasini hisoblaymiz.

$$J(\lambda s_3, y) = \left(\frac{2\pi}{\lambda s_3}\right)^2 |Hess(F)|^{-\frac{1}{2}} \times$$

$$\times\, e^{i\lambda F(x_1^c, x_2^c, s_1^c, s_2^c, s_3, y_1, y_2, y_3) + \frac{i\pi}{4} sgn\left(Hess(F(x_1^c, x_2^c, s_1^c, s_2^c))\right)} (A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + R(y, s_3, \lambda)) =$$

$$= \left(\frac{2\pi}{\lambda s_3}\right)^2 e^{i\lambda s_3 (y_3 - (1 + \Phi(y_1, y_2))} (A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + R(y, s_3, \lambda)), \qquad \lambda \to +\infty \tag{19}$$

bu yerda, $R(y, s_3, \lambda)$ qoldiq had bo‘lib ixtiyoriy $x \in U(0,0)$, $|y_1| + |y_2| < \delta$, $|y_3 - 1| < \delta$, va $|s_3| \sim 1$ va $|\lambda| > 1$ bo‘lganda $|R(y, s_3, \lambda)| \leq \frac{C}{|\lambda|}$ tengsizlik bajariladi, bunda C soni y, s_3, λ larga bog‘liq emas. Shu sababli, quyidagi

$$\left| \int_{\mathbb{R}} \chi_1(s_3)\, s_3^2 R(y, s_3, \lambda) ds_3 \right| \leq \int_{\mathbb{R}} |\chi_1(s_3)|\, |s_3^2| |R(y, s_3, \lambda)| |ds_3| \leq \frac{C}{|\lambda|} \tag{20}$$

tengsizlikka ega bo‘lamiz.

Endi, (19) dan foydalanib (14) tenglikning oxirgi tengligini quyidagicha yozamiz.

$$\frac{\lambda^3}{(2\pi)^3} \int_{\mathbb{R}} \chi_1(s_3)\, s_3^2 ds_3 \int_{\mathbb{R}^4} \chi_0(s_1 s_3, s_2 s_3) \times$$

$$\times\, a(x_1, x_2) e^{i\lambda s_3 (s_1 (y_1 - x_1) + s_2 (y_2 - x_2) + (y_3 - (1 + \Phi(x_1, x_2)))} dx_1 dx_2 ds_1 ds_2 =$$

$$= \frac{\lambda}{2\pi} \int_{\mathbb{R}} \chi_1(s_3)\, s_3^2 e^{i\lambda s_3 (y_3 - (1 + \Phi(y_1, y_2))} \times$$

$$\times\, (A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + R(y, s_3, \lambda)) ds_3 \quad \lambda \to +\infty\,.$$

Yuqoridagi tenglikdan kelib chiqqan holda, $\lambda \to +\infty$ da quyidagi tenglikka ega bo‘ldik.

$$\mu^{\lambda}(y) = \frac{\lambda}{2\pi} \int_{\mathbb{R}} \chi_1(s_3)\, s_3^2 e^{i\lambda s_3 (y_3 - (1 + \Phi(y_1, y_2))} (A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + R(y, s_3, \lambda)) ds_3 \tag{21}$$

(20) tengsizlikdan foydalangan holda yuqoridagi tenglik bilan aniqlangan integralni baholaymiz:

$$\left| \frac{\lambda}{2\pi} \int_{\mathbb{R}} \chi_1(s_3)\, s_3^2 e^{i\lambda s_3 (y_3 - (1 + \Phi(y_1, y_2))} (A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + R(y, s_3, \lambda)) ds_3 \right| \leq$$

$$\leq \left|\frac{\lambda}{2\pi}\right| \int_{\mathbb{R}} |\chi_1(s_3)|\,|s_3^2|\left|e^{i\lambda s_3(y_3-(1+\Phi(y_1,y_2))}\right| \times$$
$$\times |A(x_1^c, x_2^c, s_1^c, s_2^c, s_3) + R(y, s_3, \lambda)||ds_3| \leq C|\lambda|, \quad \lambda \to +\infty \tag{22}$$

Shunday qilib, (21) tenglikka va (22) tengsizlikka ko‘ra quyidagi bahoga ega bo‘ldik.

$$\left|\mu^{\lambda}(y)\right| \leq C|\lambda|, \quad \lambda \to +\infty$$

teorema isbotlandi.

1-eslatma: Teoremaning isbotidan kelib chiqadiki bu baho S sirtning geometrik xossalariga bog‘liq emas ekan.

Masalan,

$$\Phi(x_1, x_2) = e^{-(x_1^2+x_2^2)} - 1$$

funksiya teoremaning shartlarini qanoatlantiradi.

2-eslatma: (15) tenglikni (16) va (17) tengliklardan foydalangan holda quyidagicha yozamiz:

$$J(\lambda s_3, y) = \int_{\mathbb{R}^4} e^{i\lambda s_3 F(x_1,x_2,s_1,s_2,y_1,y_2,y_3)}\, A(x_1, x_2, s_1, s_2, s_3)dx_1dx_2ds_1ds_2 \tag{23}$$

Yuqoridagi integralning faza funksiyasi $F(x_1, x_2, s_1, s_2, y_1, y_2, y_3)$ ni $\eta = 0$ nuqtaning biror V atrofini $(x_1^c, x_2^c, s_1^c, s_2^c)$ nuqtaning shunday U atrofiga diffeomorf akslantirish orqali normal shaklga keltirish haqidagi Mors lemmasi [5, 593-b] dan foydalanib, quyidagi normal shaklda yozamiz:

$$F(\varphi(\eta, y_1, y_2, y_3), y_1, y_2, y_3) = y_3 - (1 + \Phi(y_1, y_2)) + \eta_1^2 + \eta_2^2 - \eta_3^2 - \eta_4^2 \tag{24}$$

bu yerda, $\varphi(\eta, y_1, y_2, y_3): V \to U$ diffeomorf akslantirish bo‘lib, quyidagicha aniqlangan:

$$\varphi(\eta, y_1, y_2, y_3) = \begin{pmatrix} \varphi_1(\eta_1, \eta_2, \eta_3, \eta_4, y_1, y_2, y_3) \\ \varphi_2(\eta_1, \eta_2, \eta_3, \eta_4, y_1, y_2, y_3) \\ \varphi_3(\eta_1, \eta_2, \eta_3, \eta_4, y_1, y_2, y_3) \\ \varphi_4(\eta_1, \eta_2, \eta_3, \eta_4, y_1, y_2, y_3) \end{pmatrix} \tag{25}$$

yuqoridagi akslantirishning yakobiani quyidagiga teng:

$$J\varphi(\eta, y_1, y_2, y_3) = \frac{D(\varphi(\eta, y_1, y_2, y_3))}{D(\eta_1, \eta_2, \eta_3, \eta_4)} = det \begin{Vmatrix} \frac{\partial\varphi_1}{\partial\eta_1} & \frac{\partial\varphi_1}{\partial\eta_2} & \frac{\partial\varphi_1}{\partial\eta_3} & \frac{\partial\varphi_1}{\partial\eta_4} \\ \frac{\partial\varphi_2}{\partial\eta_1} & \frac{\partial\varphi_2}{\partial\eta_2} & \frac{\partial\varphi_2}{\partial\eta_3} & \frac{\partial\varphi_2}{\partial\eta_4} \\ \frac{\partial\varphi_3}{\partial\eta_1} & \frac{\partial\varphi_3}{\partial\eta_2} & \frac{\partial\varphi_3}{\partial\eta_3} & \frac{\partial\varphi_3}{\partial\eta_4} \\ \frac{\partial\varphi_4}{\partial\eta_1} & \frac{\partial\varphi_4}{\partial\eta_2} & \frac{\partial\varphi_4}{\partial\eta_3} & \frac{\partial\varphi_4}{\partial\eta_4} \end{Vmatrix}.$$

[5, 593-b] dagi Mors lemmasini isbotlash jarayonidan kelib chiqadiki, agar $F(x_1, x_2, s_1, s_2, y_1, y_2, y_3) \in C^{N+3}(\mathbb{R}^7; \mathbb{R})$ bo‘lsa, u holda $\varphi(\eta, y_1, y_2, y_3) \in C^{N+1}(\mathbb{R}^7; \mathbb{R}^4)$ va $J\varphi(\eta, y_1, y_2, y_3) \in C^N(\mathbb{R}^7; \mathbb{R})$bo‘ladi.

Endi, (23) integralni (25) almashtirtirish orqali quyidagicha yozamiz:

$$J(\lambda s_3, y) = e^{i\lambda s_3(y_3-(1+\Phi(y_1,y_2))} \int_{\mathbb{R}^4} e^{i\lambda s_3(\eta_1^2+\eta_2^2-\eta_3^2-\eta_4^2)}\, A\big(\varphi(\eta, y_1, y_2, y_3)\big) \times$$
$$\times |J\varphi(\eta, y_1, y_2, y_3)|d\eta_1d\eta_2d\eta_3d\eta_4. \tag{26}$$

yuqoridagi tenglikning o‘ng tomonida turgan integralni Vander-Korput lemmasidan foydalanib baholaymiz. Yozuvni qisqartirish maqsadida $\tilde{A}(\eta_1, \eta_2, \eta_3, \eta_4, y_1, y_2, y_3) = A\big(\varphi(\eta, y_1, y_2, y_3)\big) \times |J\varphi(\eta, y_1, y_2, y_3)|$ belgilash kiritamiz.

$$\left| \int_{\mathbb{R}^4} e^{i\lambda s_3(\eta_1^2+\eta_2^2-\eta_3^2-\eta_4^2)}\, \tilde{A}(\eta_1, \eta_2, \eta_3, \eta_4, y_1, y_2, y_3)d\eta_1d\eta_2d\eta_3d\eta_4 \right| =$$
$$= \left| \int_{\mathbb{R}} e^{i\lambda s_3\eta_1^2} \left(\int_{\mathbb{R}^3} e^{i\lambda s_3(\eta_2^2-\eta_3^2-\eta_4^2)}\, \tilde{A}(\eta_1, \eta_2, \eta_3, \eta_4, y_1, y_2, y_3)d\eta_2d\eta_3d\eta_4 \right) d\eta_1 \right| \leq$$

$$\leq \frac{C_1}{\lambda^{\frac{1}{2}}} \int_{\mathbb{R}} \left| \int_{\mathbb{R}^3} e^{i\lambda s_3(\eta_2^2-\eta_3^2-\eta_4^2)} \frac{\partial \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1} d\eta_2 d\eta_3 d\eta_4 \right| d\eta_1 \tag{27}$$

$$\left| \int_{\mathbb{R}^3} e^{i\lambda s_3(\eta_2^2-\eta_3^2-\eta_4^2)} \frac{\partial \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1} d\eta_2 d\eta_3 d\eta_4 \right| =$$

$$= \left| \int_{\mathbb{R}} e^{i\lambda s_3 \eta_2^2} \left(\int_{\mathbb{R}^2} e^{i\lambda s_3(-\eta_3^2-\eta_4^2)} \frac{\partial \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1} d\eta_3 d\eta_4 \right) d\eta_2 \right| \leq$$

$$\leq \frac{C_2}{\lambda^{\frac{1}{2}}} \int_{\mathbb{R}} \left| \int_{\mathbb{R}^2} e^{i\lambda s_3(-\eta_3^2-\eta_4^2)} \frac{\partial^2 \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1 \partial \eta_2} d\eta_3 d\eta_4 \right| d\eta_2 \tag{28}$$

$$\left| \int_{\mathbb{R}^2} e^{i\lambda s_3(-\eta_3^2-\eta_4^2)} \frac{\partial^2 \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1 \partial \eta_2} d\eta_3 d\eta_4 \right| =$$

$$= \left| \int_{\mathbb{R}} e^{i\lambda s_3(-\eta_3^2)} \left(\int_{\mathbb{R}} e^{i\lambda s_3(-\eta_4^2)} \frac{\partial^2 \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1 \partial \eta_2} d\eta_4 \right) d\eta_3 \right| \leq$$

$$\leq \frac{C_3}{\lambda^{\frac{1}{2}}} \int_{\mathbb{R}} \left| \int_{\mathbb{R}} e^{i\lambda s_3(-\eta_4^2)} \frac{\partial^3 \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1 \partial \eta_2 \partial \eta_3} d\eta_4 \right| d\eta_3 \tag{29}$$

$$\left| \int_{\mathbb{R}} e^{i\lambda s_3(-\eta_4^2)} \frac{\partial^3 \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1 \partial \eta_2 \partial \eta_3} d\eta_4 \right| \leq$$

$$\leq \frac{C_4}{\lambda^{\frac{1}{2}}} \int_{\mathbb{R}} \left| \frac{\partial^4 \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1 \partial \eta_2 \partial \eta_3 \partial \eta_4} \right| d\eta_4 \tag{30}$$

Endi, (30), (29), (28) , (27) tengsizliklardan va (26) tenglikdan foydalanib, $J(\lambda s_3, y)$ integral uchun quyidagi tengsizlikka ega bo‘lamiz:

$$|J(\lambda s_3, y)| \leq \frac{\tilde{C}}{\lambda^{-2}} \int_{\mathbb{R}^4} \left| \frac{\partial^4 \tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3)}{\partial \eta_1 \partial \eta_2 \partial \eta_3 \partial \eta_4} \right| d\eta_1 d\eta_2 d\eta_3 d\eta_4. \tag{31}$$

Yuqoridagi (31) tengsizlik va (15), (14) tengliklarga ko‘ra quyidagiga ega bo‘ldik.

$|\mu^{\lambda}(y)| \leq C|\lambda|, \quad \lambda \to +\infty$

Yuqorida xulosa qildikki, agar faza funksiyasi $C^7(\mathbb{R}^7;\mathbb{R})$ sinfga tegishli bo‘lsa, uning yakobiani $C^4(\mathbb{R}^7;\mathbb{R})$ sinfga tegishli bo‘ladi. Agar amplituda funksiyasi $A(x_1,x_2,s_1,s_2,s_3) \in C^4(\mathbb{R}^5;\mathbb{R})$ bo‘lsa va faza funksiyasi $F(x_1,x_2,s_1,s_2,y_1,y_2,y_3) \in C^7(\mathbb{R}^7;\mathbb{R})$ bo‘lsa, u holda $\tilde{A}(\eta_1,\eta_2,\eta_3,\eta_4,y_1,y_2,y_3) \in C(\mathbb{R}^4;\mathbb{R})$ bo‘ladi. Bu holda ham teoremada keltirilgan baho o‘rinli bo‘lar ekan.

Xulosa qilib, shuni aytishimiz mumkinki, teoremada faza funksiyasi yetti marta uzluksiz differensiallanuvchi, ampituda funksiyasi esa to‘rt marta uzluksiz differensiallanuvchi bo‘lishi yetarlidir.

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