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САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ ШАРОФА РАШИДОВА

SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

ILMIY AXBOROTNOMA

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DENSITY AND REGULAR DISTRIBUTION

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Abstract. It is known that the fields of number theory and mathematical analysis are very important. In this article, we will study the close part of these two fields and analyse some problems of practical importance. We study the character of the location of sequences of the form in an interval by applying strong theorems.

Keywords: Kronecker theorem, Weil criterion, consequences, Dirichlet theorems, continuous functions.

Плотность и регулярное распределение

Аннотация. Известно, что области теории чисел и математического анализа очень важны. В этой статье мы изучим тесную часть этих двух областей и проанализируем некоторые проблемы, имеющие практическое значение. Мы изучаем характер расположения последовательностей вида в интервале, применяя сильные теоремы.

Ключевые слова: теорема Кронекера, критерий Вейля, следствия, теоремы Дирихле, непрерывные функции.

Zichlik va tekis taqsimlanish

Annotatsiya. Ma`lumki sonlar nazariyasi va Matematik analiz sohalari o‘ta muhim ahamiyatga ega. Biz ushbu maqolada bu ikki sohaning bir biriga yaqin bo‘lgan qismini o‘rganamiz va bir qancha amaliy ahamiyatga ega bo‘lgan masalalarni tahlil qilamiz. Oldindan berilgan turli xil ko‘rinishdagi ketma-ketliklarni biror intervalda joylashuv xarakterini kuchli teoremalarni tatbiq etgan holda o‘rganib chiqamiz.

Kalit so’zlar: Kroneker teoremasi,Veil kriteriyasi, ketma-ketliklar, Dirixle teoremasi, uzluksiz funksiyalar.

Introduction. Everyone knows that $(\{na\})_{n\geq 1}$ is dense in $[0,1]$ if a is an irrational number, a classical theorem of Kronecker. Various applications of this nice result have appeared in different contests and will probably be the object of Olympiad problems in the future. However, there are some examples in which this result is ineffective. A simple one is as follows: using Kronecker’s theorem, one easily proves that for any positive integer a that is not a power of 10, there exists n such that a^n begins with 2006.

The natural question of what fraction of numbers between 1 and n have this property is much more difficult, and to answer it, we need some stronger tools. This is the reason for which we will try to discuss some classical approximation theorems, particularly the very efficient. Weil criterion and its consequences. The proofs are nontrivial and require some heavy-duty analysis. However, the consequences that will be discussed here are almost elementary.

Of course, one cannot start a topic about approximation theorems without talking first about Kronecker’s theorem. We skip the proof, not only because it is very well known but also

because we will prove a much stronger result about the sequence$(\{na\})_{n\geq 1}$. Instead, we will discuss two beautiful problems that are consequences of this theorem.

Example 1: Prove that the sequences $\left(\left\{n\sqrt{2003}\right\}\right)_{n\geq 1}$contain arbitrarily long geometric sequences with a large ratio.

Radu Gologan, IMO TST Romania

Solution: Let us take p a very large number. We will prove that there are arbitrarily long geometric sequences with ratio. Given $n \geq 3$, let us prove that we can find a positive integer m such that

$$\left[p^k m\sqrt{2003}\right] = p^k\left[m\sqrt{2003}\right] \qquad \text{for all} 1 \leq k \leq n$$

If the existence of such a number is proven, then the conclusion is immediate.However, observe that$\left[p^k m\sqrt{2003}\right] = p^k\left[m\sqrt{2003}\right]$ is equivalent to $\left[p^k\left\{m\sqrt{2003}\right\}\right] = 0$ or to $\left\{m\sqrt{2003}\right\} < \frac{1}{p^n}$. The existence of a positive integer m with the last property is ensured by Kronecker's theorem.

We will consider the following famous Dirichlet theorems.

Example 2 (Dirichlet) Let $x_1, x_2, x_3, , , , , , x_{k-1}, x_k$ be some real numbers and let $\varepsilon > 0$. There exists a positive integer n and integers $p_1, p_2, p_3, , , , , , p_{k-1}, p_k$ such that $|nx_i - p_i| < \varepsilon$ for all i.

Solution: Thus, we need to prove that if we have a finite set of real numbers, we can multiply all its elements of the new set as close to integers as we want.

Let us choose an integer $N > \frac{1}{\varepsilon}$ and partition the interval $[0,1]$ into N intervals,

$[0,1] = \cup_{s=1}^{N} J_s, \qquad J_s = \left[\frac{s-1}{N}, \frac{s}{N}\right].$

Now, choose $n = N^k + 1$ and associate to any positive integer $q \in \{1,2, \ldots., n\}$ a sequence of k positive integers $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots \alpha_k$ where $\alpha_2 = s$ if and only if $\{qx_i\} \in J_s$.

We obtain at most N^k sequences corresponding to these numbers, so by using Dirichlet's criterion, we can find

$1 \leq u < v \leq n$

such that the same sequence is associated with $u \; and \; v$.

This means that for all $1 \leq i \leq k$, we have

$$|\{ux_i\} - \{vx_i\}| < \frac{1}{N} \leq \varepsilon$$

It suffices thus to pick$n = v - u, \; p_i = [vx_i] - [ux_i]$.

Here is how we can use this result in problems where it is more comfortable to work with integers. However, don't kid yourself, there are few such problems. The one we are going to discuss has had a circuitous itinerary between world's Olympiads: proposed for IMO long time ago, it appeared next at the W.L. Putnam Competition and later on in a Japanese Mathematical Olympiad.

Example 3: Let $x_1, x_2, x_3, , , , , , , x_{2n+1}$ be positive real numbers with the following property: for any $1 \leq i \leq 2n+1$, one can make two groups of n numbers by using

x_j , $j \neq i$, such that the sum of the numbers in each group is the same. Prove that all numbers are equal.

Solution: Of course, the problem for integers is very well known and easy: it suffices to observe that in this case, all numbers x_i have the same parity and the use of infinite descent solves the problem (either they are all even and in this case, we divide each one by two and obtain a new

set with a smaller sum and properties; otherwise, we subtract 1 from each one and then divide by 2).

Now, assume that they are real numbers, which is clearly much more subtle. First, if they are all rational, it suffices to multiply by their common denominator and apply the first case. Thus, assume that at least one of the numbers is irrational.

Consider $\varepsilon > 0$, a positive integer n, and some integers $p_1, p_2, p_3, , , , , , p_{k-1}, p_k$ such that $|nx_i - p_i| < \varepsilon$ for all i. We claim that if $\varepsilon > 0$ is small enough, the corresponding $p_1, p_2, p_3, , , , , , p_{k-1}, p_k$ has the same property as c. Indeed, take some i and write the partition condition in the form

$$\sum_{j\neq i} a_{ij}\, nx_j = 0 \quad \text{or} \quad \sum_{j\neq i} a_{ij}(nx_j - p_j) = -\sum_{j\neq i} a_{ij}(p_j)$$

where of course $a_{ij} \in \{-1,1\}$

then

$$\left|\sum_{j\neq i} a_{ij}(p_j)\right| = \left|\sum_{j\neq i} a_{ij}(nx_j - p_j)\right| \leq 2n\varepsilon$$

Thus if we choos $\varepsilon < \frac{1}{2n}$, then $\sum_{j\neq i} a_{ij}(p_j) = 0$ and so $p_1, p_2, p_3, , , , , , p_{k-1}, p_k$ have the same property. Since they are all integers, they must be equal.

Thus, we have proved that for any $N > 2n$, there are integers n_N , p_N such that

$$|n_N x_i - p_N| \leq \frac{1}{N}$$

Because at least one of the numbers $x_1, x_2, x_3, , , , , , , x_{2n+1}$ is irrational, it is not difficult to prove that the sequence $(n_N)_{N>2n}$ is unbounded.

However, $|n_N| \max_{i,j}|x_i - x_j| < \frac{2}{N}$; hence, $\max_{i,j}|x_i - x_j| = 0$, and the problem is solved.

Now, let us turn to more quantitative results about the set of fractional parts of natural multiples of different real numbers. The following criterion, due to Weil, is famous and deserves to be discussed because of its beauty and apparent simplicity.

Weil criterion. Let $(a_n)_{n\geq 1}$ be a sequence of real numbers from the interval $[0,1]$. Then, the following statements are equivalent:

a) For any real numbers $0 \leq a \leq b \leq 1$,

$$\lim_{n\to\infty} \frac{|\{i | 1\leq i\leq n, a_i \in [a,b]\}|}{n} = b - a$$

b) For any continuous function $f: [0,1] \to R$,

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} f(a_k) = \int_0^1 f(x)dx$$

c) For any positive integers $p \geq 1$,

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} e^{2i\pi p a_k} = 0.$$

In this case, we will say that the sequence is equidistributed.

We will present just a sketch of the solution, containing yet all the necessary ingredients.

First, we observe that a) says precisely that b) is true for the characteristic function of any subinterval of $[0,1]$. By linearity, this remains true for any piecewise function. Now, there is a well-known and easy-to-verify property of continuous functions: they can be uniformly approximated with piecewise functions.

That is, given $\varepsilon > 0$,, we can find a piecewise function g such that

$$|g(x) - f(x)| < \varepsilon \text{ for all } x \in [0,1].$$

However, then if we write

$$\left|\frac{1}{n}\sum_{k=1}^{n} f(a_k) - \int_0^1 f(x)dx\right|$$

$$\leq \frac{1}{n}\sum_{k=1}^{n}|f(a_k) - g(a_k)| + \int_0^1 |f(x) - g(x)|dx + \left|\frac{1}{n}\sum_{k=1}^{n} g(a_k) - \int_0^1 g(x)dx\right|$$

and apply the result in b) for the function g,, we easily deduce that b) is true for any continuous function.

The fact that b) implies c) is immediate. More subtle is that b) implies a).

Let us consider the subinterval $I = [a, b]$ with $0 < a < b < 1$.

Next, consider two sequences of continuous functions f_k, g_k such that f_k is zero on $[0, a], [b, 1]$ and 1 on $\left[a + \frac{1}{k}; b - \frac{1}{k}\right]$ (being affine otherwise), while f_k has "the same" properties but is greater than or equal to δ_k

Therefore,

$\frac{1}{n}\sum_{j=1}^{n} f_k(a_j) \leq \frac{|\{i|1\leq i\leq n, a_i \in [a,b]\}|}{n} \leq \frac{1}{n}\sum_{j=1}^{n} g_k(a_j)$

However, from the hypothesis

$\frac{1}{n}\sum_{j=1}^{n} f_k(a_j) \to \int_0^1 f_k(x)dx = b - a - \frac{1}{k}$

And

$$\frac{1}{n}\sum_{j=1}^{n} g_k(a_j) \to \int_0^1 g_k(x)dx = b - a + \frac{1}{k}$$

Now, let us take $\varepsilon > 0$ and k to be sufficiently large. The above inequalities show that for all sufficiently large positive integers n ,

$\left|\frac{|\{i|1\leq i\leq n, a_i \in [a,b]\}|}{n} - b + a\right| \leq 2\varepsilon$

and the conclusion follows. The reader has already seen how to adapt this proof for the case$a = 0 \; or \; b = 1$.

Finally, let us prove that c) implies b). Of course, a linearity argument allows us to assume that b) is true for any trigonometric polynomial of any degree. Because any continuous function $f : [0, 1] \to R$ satisfying $f(0) = f(1)$ can be uniformly approximated by trigonometric polynomials (this is a truly nontrivial result due to Weierstrass), we deduce that b) is true for continuous functions f for which $f(0) = f(1)$. Now, given $f : [0, 1] \to R$ continuous, it is immediate that for any $\varepsilon > 0$ we can find two continuous functions g, h, both having equal values at 0 and 1 and such that

$$|f(x) - g(x)| \leq h(x) \quad \text{and} \int_0^1 h(x)dx \leq \varepsilon$$

Using the same arguments as those used to prove that b) implies a), one can easily see that b) is true for any continuous function.

The first problem that we discuss is in fact the most common result about equidistribution. We invite the reader to find an elementary proof to appreciate the power of Weil's criterion.

Before presenting the second problem, we need another definition:

we say that the sequence $(a_n)_{n\geq 1}$is uniformly distributed $mod\, 1$ if the sequence of fractional parts of an is equidistributed.

Therefore, here is the classical example.

Example 5. Let a be an irrational number. Then, the sequence $(na)n \geq 1$ is uniformly distributed $mod\ 1$.

Solution. Well, after so much work, we deserve a reward: this is a simple consequence of Weil's criterion. Indeed, it suffices to prove that c) is verified, which reduces to proving that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} e^{2i\pi pak} = 0 \qquad (*)$$

for all integers $p \geq 1$.

However, this is just a geometric series. A one-line computation shows that $(*)$ is trivially satisfied, and thus, we have the desired result.

We quote the famous Vander der Carput theorem without proof and give a problem in which it can be applied at the end of the article.

(Van der Corput) Let (x_n) be a sequence of real numbers such that the sequences $(x_{n+p} - x_n)n \geq 1$ are equidistributed for all $p \geq 1$. Then, the sequence (x_n) is also equidistributed.

Example 6. Let f be a polynomial with real coefficients and an irrational leading coefficient. Then, the sequence $(\{f(n)\})n \geq 1$ is equidistributed.

The reader has probably noticed that this is an immediate consequence of Van der Corput's theorem (but just imagine the amount of work done to arrive at this conclusion!!!). Indeed, the proof by induction is immediate. Indeed, if f has degree 1, then the conclusion is immediate (see example 5). Now, if the result holds for polynomials of degree at most k, it suffices (by Van der Corput's theorem) to prove that for all positive integers p, the sequence $(f(n + p) - (f(n))$ is equidistributed. However, this is exactly the induction hypothesis applied to the polynomial (whose leading coefficient is clearly irrational) $f(X + p) - f(X)$. The proof by induction finishes here.

At the end of the article, I would like to mention one great issue for independent work

Here is a problem that is apparently very difficult, but which is again a simple consequence of Kronecker's theorem.

Example 7. Consider $k \geq 1$ and a such that $\log a$ is irrational. Define the sequence x_n as the number formed by the first k digits of the number $[a^n]$ with $n \geq 1$. Prove that this sequence is not eventually periodical.

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