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ON THE PROPERTIES OF SOLUTIONS OF A NONLINEAR FILTRATION PROBLEM WITH A SOURCE AND MULTIPLE NONLINEARITIES

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Abstract. In this work, we studied the conditions for global solvability and unsolvability of a nonlinear filtration problem with a source and with a nonlinear boundary flow. The critical exponent of the global existence of the solution and the critical exponent of the Fujita type of the problem of nonlinear filtration in an inhomogeneous medium are established. An asymptotic representation of a compactly supported solution is obtained, which makes it possible to carry out a numerical experiment.

Key words: blow-up; nonlinear boundary condition; nonlinear boundary flux; critical global existence curve; degenerate parabolic systems; critical exponents of the Fujita type.

О свойствах решений нелинейной задачи фильтрации с источником и кратными нелинейностями

Аннотация. В работе исследованы условия глобальной разрешимости и неразрешимости нелинейной задачи фильтрации с источником и с нелинейным пограничным потоком. Установлены критический показатель глобального существования решения и критический показатель типа Фуджиты задачи нелинейной фильтрации в неоднородной среде. Получено асимптотическое представление финитного решения, позволяющее провести численный эксперимент.

Ключевые слова: режим с обострением; нелинейное граничное условие; нелинейный граничный поток; критическая глобальная кривая существования; вырожденные параболические системы; критические экспонента типа Фуджита.

Manbaga ega bo’lgan nochiziqli chegaraviy shart bilan berilgan nochiziqli filtrlash masalasi yechimlarining xossalari haqida.

Annotatsiya. Nochiziqli filtrlash muammosining manba va chiziqli chegara oqimi bilan global yechilishi va yechilmasligi shartlari o‘rganilgan. Yechimning global mavjudligining tanqidiy ko'rsatkichi va bir hil bo'lmagan muhitda chiziqli bo'lmagan filtrlash muammosining Fujita tipidagi kritik ko'rsatkichi o'rnatildi. Cheklangan yechimning asimptotik tasviri olindi, bu esa sonli tajribani amalga oshirish imkonini beradi.

Kalit so'zlar: filtrlash, global yechimlar, portlatish, kritik egri chiziq, asimptotiklar, raqamli tahlil.

1. Introduction. We will discuss the next parabolic equation

$$\rho(x)\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(\left|\frac{\partial u^m}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}\right)+\rho(x)u^{\beta},\quad (x,t)\in R_+\times(0,+\infty) \tag{1}$$

with nonlinear boundary condition

$$-\left|\frac{\partial u^m}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}(0,t)=u^q(0,t),\ t>0 \tag{2}$$

and initial value condition

$$u(x,0)=u_0(x)\geq 0,\qquad x\in R_+ \tag{3}$$

where $p>1+1/m,\ \beta,q>0,\ \rho(x)=x^{-n},\ n\in R,\ u_0(x)$ – bounded, continuous, non-negative and non-trivial initial data.

Equation (1) occurs in various areas of natural science [1, 3–5]. For example, equation (1) is considered in mathematical modeling of the thermal conductivity of nanofluids, in the study of problems of fluid flow through porous media, in problems of the dynamics of biological populations, polytropic filtration, structure formation in synergetics and nanotechnologies, and in a number of other areas [1–4].

Equation (1) is called a parabolic equation with variable density [1] and in case $m(p-1)-1>0$ corresponds to the equation of slow filtration [2-3]. Problem (1)–(3) has been intensively studied by many authors (see [2, 6–17] and references therein) for various values of numerical parameters.

In [17], the authors, considering problem (1)-(3) in the case $m=1,\ \rho(x)=1$, proved that for $0<\beta\le 1$ and $q\le(p-1)(2-n)/(p-n)$ any non-trivial solution of problem (1)-(3) is global. If $\beta<1$ and $q>(p-1)(2-n)/(p-n)$, then each solution of problem (1)-(3) is unbounded in a finite time.

In work [5], the condition of global unsolvability in time of the solution of the Cauchy problem for equation (1) at was obtained that $m=1,\ \rho(x)=1$ and the critical exponent of the Fujita type $\beta=2p-1$ was established.

Some properties of solutions to problem (1)–(3) at $\rho(x)=1,\ \ m=1$ were studied in [9]. They obtained the critical exponent of the global existence of the solution and the critical exponent of the Fujita type by constructing the sub and super solutions.

In [7], the unboundedness of the solution of the following reaction-filtration model with a nonlinear boundary condition was studied

$$u_t=\Delta u^m+u^\beta,\quad (x,t)\in\Omega\times(0,T),$$
$$\frac{\partial u}{\partial\eta}=u^q,\quad (x,t)\in\partial\Omega\times(0,T),$$
$$u(x,0)=u_0(x),\quad x\in\Omega,$$

where $\Omega\in R^N$ is the bounded area. The authors showed that all positive solutions exist globally in the case $m>1$ if and only if $\beta, q\le 1$, and in the case $m\le 1$ when $\beta\le 1,\ q\le 2m/(m+1)$.

As is known, degenerate equations may not have classical solutions. Therefore, its solution is understood in a generalized sense.

Definition 1. A function is called a weak solution to problem (1)-(3) at $\Omega=\{R_+\times(0,T)\}$, if $0\le u(x,t)\in C(\Omega),\ \left|\frac{\partial u^m}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}\in C(\Omega)$, and if it satisfies (1)–(3) in a generalized sense at Ω, where $T>0$ is the maximum lifetime.

2. Main results

Below, we will determine the condition of solvability and unsolvability in general in terms of time for solving problem (1)–(3) in the case of slow filtration. It is assumed that $p>1+1/m$.

Theorem 1. *If* $q\le\frac{m(1-n)+1}{p-n}(p-1)$ *and* $0<\beta\le 1$ *, then any solution to problem (1)–(3) is global.*

Proof. We are looking for a globally time-defined super solution in the following self-similar form

$$\bar u=e^{Bt}(D+e^{-L\xi})^{\frac{1}{m}},\ \xi=(1+x)e^{-Jt}$$

where $D>\|u_0\|_\infty$, $L^{\frac{m(p-1)}{q}}e^{-\frac{Lm(p-1)}{q}}-e^{-L}=D$, $B=((p-1)L^p+(D+1)^{\frac{\beta}{m}})/D^{\frac{1}{m}}$, $J=B(m(p-1)-1)/(p-n)$

After some calculations we have:

$$\rho(x)\frac{\partial\bar u}{\partial t}=\xi^{-n}e^{(B-nJ)t}\cdot\left[B(D+e^{-L\xi})^{\frac{1}{m}}+\frac{JL}{m}e^{-L\xi}(D+e^{-L\xi})^{\frac{1-m}{m}}\xi\right]$$

$$\frac{\partial}{\partial x}\left(\left|\frac{\partial \bar{u}^m}{\partial x}\right|^{p-2}\frac{\partial \bar{u}^m}{\partial x}\right)=e^{[(mB-J)(p-1)-J]t}(p-1)\cdot L^p e^{-L\xi(p-1)},$$

$$\rho(x)\bar{u}^{\beta}=\xi^{-n}e^{(\beta B-Jn)t}(D+e^{-L\xi})^{\frac{\beta}{m}},$$

$$-\left|\frac{\partial \bar{u}^m}{\partial x}\right|^{p-2}\frac{\partial \bar{u}^m}{\partial x}(0,t)=e^{(mB-J)(p-1)t}\cdot L^{p-1}e^{-Le^{-Jt}(p-1)}.$$

Now let's show that the function $\bar{u}(x,t)$ is the super solution of problem (1)-(3). According to the principle of comparison, it must satisfy the following inequality:

$$\rho(x)\frac{\partial u}{\partial t}\geq\frac{\partial}{\partial x}\left(\left|\frac{\partial u^m}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}\right)+\rho(x)u^{\beta},\qquad (x,t)\in R_+\times(0,+\infty) \tag{4}$$

$$-\left|\frac{\partial \bar{u}^m}{\partial x}\right|^{p-2}\frac{\partial \bar{u}^m}{\partial x}(0,t)\geq\bar{u}^q(0,t),\qquad t>0 \tag{5}$$

It is easy to check that if $0<\beta\leq 1$ and $0<q\leq(m(1-n)+1)(p-1)/(p-n)$ by the definition of *B, L, D, J,* inequalities (4) and (5) hold. From here $\bar{u}(x,0)\geq u_0(x)$ and $\bar{u}(0,0)\geq u_0(0)$. Thus, Theorem 1 is proved by the comparison principle.

Remark 1. *Theorem 1 shows that the critical exponent of the global existence of a solution to problem (1)–(3) is equal to*

$$\left\{\beta=1,\ 0<q\leq\frac{(2-n)(p-1)}{p-n}\right\}\cup\left\{q=\frac{(2-n)(p-1)}{p-n},\ \beta\leq 1\right\}$$

Theorem 2. *If* $\beta<1$ *and* $q\geq\dfrac{(m(1-n)+1)(p-1)}{(p-n)}$, *then the solution of problem (1)–(3) is unbounded in a finite time.*

Proof. We will look for an unbounded sub solution in the following self-similar form:

$$u_{-}=t^{\sigma}\varphi(\eta),\quad \eta=xt^{-\gamma} \tag{6}$$

where $\sigma=1/(1-\beta)$, $\gamma=(m(p-1)-\beta)/(1-\beta)(p-n)$ and $\varphi(\eta)$ is a solution to the following problem

$$\frac{d}{d\eta}\left(\varphi^{(m-1)(p-1)}\left|\frac{d\varphi}{d\eta}\right|^{p-2}\frac{d\varphi}{d\eta}\right)+\frac{\gamma\eta^{1-n}}{m^{p-1}}\frac{d\varphi}{d\eta}-\frac{\eta^{-n}}{m^{p-1}}[\sigma\varphi-\varphi^{\beta}]=0 \tag{7}$$

$$-m^{p-1}\varphi^{(m-1)(p-1)}\left|\frac{d\varphi}{d\eta}\right|^{p-2}\frac{d\varphi}{d\eta}(0)=\varphi^q(0). \tag{8}$$

To be $u_{-}(x,t)$ the sub solution to problem (1)-(3), the function must satisfy the following inequalities:

$$\frac{d}{d\eta}\left(\varphi^{(m-1)(p-1)}\left|\frac{d\varphi}{d\eta}\right|^{p-2}\frac{d\varphi}{d\eta}\right)+\frac{\gamma\eta^{1-n}}{m^{p-1}}\frac{d\varphi}{d\eta}-\frac{\eta^{-n}}{m^{p-1}}[\sigma\varphi-\varphi^{\beta}]\geq 0 \tag{9}$$

$$-m^{p-1}\varphi^{(m-1)(p-1)}\left|\frac{d\varphi}{d\eta}\right|^{p-2}\frac{d\varphi}{d\eta}(0)\leq\varphi^q(0) \tag{10}$$

Let

$$\varphi(\eta)=E\left(a-\eta^{\frac{p-n}{p-1}}\right)_{+}^{\frac{p-1}{m(p-1)-1}} \tag{11}$$

where E, a are constants, which are defined below. For $\varphi(\eta)$ establish the following relations

$$\varphi'(\eta)=\frac{n-p}{m(p-1)-1}E\eta^{\frac{1-n}{p-1}}\left(a-\eta^{\frac{p-n}{p-1}}\right)^{\frac{p-m(p-1)}{m(p-1)-1}},$$

$$\varphi^{(m-1)(p-1)}\left|\frac{d\varphi}{d\eta}\right|^{p-2}\frac{d\varphi}{d\eta}=\left(\frac{n-p}{m(p-1)-1}\right)^{p-1}\cdot E^{m(p-1)}\eta^{1-n}\left(a-\eta^{\frac{p-n}{p-1}}\right)^{\frac{p-1}{m(p-1)-1}},$$

$$\varphi^{(m-1)(p-1)}\left|\frac{d\varphi}{d\eta}\right|^{p-2}\frac{d\varphi}{d\eta}(0)=0$$

Taking the above relations to (9), (10) we obtain the inequality $a^{\frac{(p-1)(\beta-1)}{m(p-1)-1}}\geq\left(\gamma(1-n)+\sigma\right)E^{1-\beta}$ in which it is easy to verify that if $\beta<1$, then (9) and (10) hold. Thus, $u_(x,t)$ is the sub solution of problem (1)-(3) for any non-trivial initial data.

Theorem 3. *If* $\beta>2p-1$ *and* $q<(p-1)(m-mn+1)/(p-n)$, *then the solution of problem (1)–(3) is unbounded in a finite time.*

Proof. In this case, we will prove that, by the initial condition, the solution will be large enough, and is included in the set of initial data, for which one reaction term is enough to cause an "explosion". Consider the sub self-similar solution of problem (1)–(3) without a source:

$$u_b(x,t)=t^{\alpha}g(\xi),\quad \xi=xt^{-\gamma},\ \rho(x)=x^{-n} \tag{12}$$

where $\alpha=\frac{p-1}{(p-1)(m-mn+1)-q(p-n)}$, $\gamma=\frac{m(p-1)-q}{(p-1)(m-mn+1)-q(p-n)}$. Let us show that the function defined by formula (12) is the sub solution. Then, according to the comparison principle, the function $u_b(x,t)$ must satisfy the following inequalities:

$$\frac{d}{d\xi}\left(g^{(m-1)(p-1)}\left|\frac{dg}{d\xi}\right|^{p-2}\frac{dg}{d\xi}\right)+\frac{\gamma\xi^{1-n}}{m^{p-1}}\frac{dg}{d\xi}-\frac{\xi^{-n}}{m^{p-1}}\alpha g\geq 0, \tag{13}$$

$$-m^{p-1}g^{(m-1)(p-1)}\left|\frac{dg}{d\xi}\right|^{p-2}\frac{dg}{d\xi}(0)\leq g^{q}(0) \tag{14}$$

Let us set

$$\overline{g}(\xi)=B\left(b-\xi^{\frac{p-n}{p-1}}\right)_{+}^{\frac{p-1}{m(p-1)-1}}$$

where b and B are positive constants, which are defined below. After some calculations we have

$$g'(\xi)=\frac{n-p}{m(p-1)-1}B\cdot\xi^{\frac{1-n}{p-1}}\left(b-\xi^{\frac{p-n}{p-1}}\right)_{+}^{\frac{(p-1)(1-m)+1}{m(p-1)-1}}$$

$$-m^{p-1}g^{(m-1)(p-1)}\left|\frac{dg}{d\xi}\right|^{p-2}\frac{dg}{d\xi}=\left(\frac{n-p}{m(p-1)-1}\right)^{p-1}B^{m(p-1)}\cdot\xi^{1-n}\left(b-\xi^{\frac{p-n}{p-1}}\right)^{\frac{2(p-1)(1-m)+1}{m(p-1)-1}(p-1)}$$

Noting, that

$$-\frac{\xi^{-n}}{m^{p-1}}(1-n)\gamma g-\alpha m^{1-p}\xi^{-n}g=-m^{1-p}\xi^{-n}g(\gamma(1-n)+\alpha)\geq 0,\quad m^{1-p}\xi^{-n}g>0$$

we obtain $\gamma(1-n)+\alpha\leq 0$. Calculating the inequality, we determine $q<(p-1)(m-mn+1)/(p-n)$ what makes it easy to check the validity of (13) and (14). It follows from the comparison principle that the solution to problem (1)–(3) is unbounded.

Theorem 4. If $\beta>1$ and $q<(p-1)(m-mn+1)$, then any solution of problem (1)-(3) is unlimited in time.

Remark 2. It is easy to see from Theorems 3 and 4 that the Fujita-type critical exponent of problem (1)-(3) is equal to $\{\beta=2p-1,\ q\ge(m+1)(p-1)\}\cup\{\beta\ge 2p-1,\ q=(m+1)(p-1)\}$

Theorem 5. If $\beta>((p-n)q-p+1)/(p-1)$ and $q>(p-1)(m-mn+1)$, then the solution to problem (1)–(3) is global in time.

Theorems 4 and 5 are proved in the same way as in [13, 16].

Now let us show the asymptotics of the self-similar solution of problem (1)-(3).

Consider the following self-similar solution to problem (1)–(3):

$$u_1(x,t)=t^{\alpha}\varphi(\xi),\quad \xi=xt^{-\gamma} \tag{15}$$

where $\alpha=1/(1-\beta)$ and $\gamma=(m(p-1)-\beta)/(1-\beta)(p-n)$, $\varphi(\xi)$ is a solution to the following problem

$$\frac{d}{d\xi}\left(\varphi^{(m-1)(p-1)}\left|\frac{d\varphi}{d\xi}\right|^{p-2}\frac{d\varphi}{d\xi}\right)+\frac{\gamma\xi^{1-n}}{m^{p-1}}\frac{d\varphi}{d\xi}-\frac{\xi^{-n}}{m^{p-1}}[\alpha\varphi-\varphi^{\beta}]=0 \tag{16}$$

$$-m^{p-1}\varphi^{(m-1)(p-1)}\left|\frac{d\varphi}{d\xi}\right|^{p-2}\frac{d\varphi}{d\xi}(0)=\varphi^{q}(0). \tag{17}$$

Consider the function

$$\overline{\varphi}(\xi)=\left(a-\left(-\frac{\gamma}{m^{p-1}}\right)^{\frac{1}{p-1}}\frac{m(p-1)-1}{p-n}\xi^{\frac{p-n}{p-1}}\right)_{+}^{\frac{p-1}{m(p-1)-1}} \tag{18}$$

where $a>0$, $(y)_{+}=\max(0,y)$. Let us show that function (18) is the asymptotics of solutions to problem (16), (17).

Theorem 6. Solution of problem (16), (17) with compact support has the asymptotics

$\varphi(\xi)=\overline{\varphi}(\xi)(1+o(1))$

when $\xi\to\left(\dfrac{a(p-n)m}{(m(p-1)-1)(-\gamma)^{1/(p-1)}}\right)^{\frac{p-1}{p-n}}$.

Proof. We will look for a solution to equation (16) in the following form

$\varphi=\overline{\varphi}(\xi)\omega(\tau)$ (19)

with $\tau=-\ln(a-b\xi^{\frac{p-n}{p-1}})$, $b=(-\gamma)^{\frac{1}{p-1}}\dfrac{m(p-1)-1}{m(p-n)}$, where $\tau\to+\infty$ for $\xi\to(a/b)^{\frac{p-1}{p-n}}$.

Substituting (19), taking into account (18), into equation (16), we obtain the form:

$$\frac{d}{d\tau}(L_2w)^{p-1}+\left(k_1\phi_1(\tau)+\frac{1-p}{m(p-1)-1}\right)(L_2w)^{p-1}+k_2L_1w-k_3\phi_1(\tau)w-k_4\phi_2(\tau)w^{\beta}=0 \tag{20}$$

where $L_1w=\dfrac{w}{m(p-1)-1}-\dfrac{w'}{p-1}$, $L_2w=\dfrac{w^m}{m(p-1)-1}-\dfrac{w^{(m-1(p-1)}}{p-1}w'$,

$\phi_1(\tau)=\dfrac{e^{-\tau}}{a-e^{-\tau}}$, $\phi_2(\tau)=e^{-\frac{(p-1)(\beta+m)-p}{m(p-1)-1}\tau}/(a-e^{-\tau})$, $k_1=\dfrac{(1-p)(1-n)}{n-p}$,

$k_2=\dfrac{\gamma[b(p-n)]^{1-p}(p-1)}{m^{p-1}}$, $k_3=\dfrac{\alpha(p-1)}{m^{p-1}b^{p-1}(p-n)^p}$, $k_4=\dfrac{p-1}{m^{p-1}b^{p-1}(p-n)^p}$

Note that the study of solutions of the last equation is equivalent to the study of those solutions of equation (1), each of which satisfies the inequalities on some interval $[\tau_0,+\infty)$:

$$w(\tau)>0,\ \frac{w(\tau)}{p-2}-\frac{w'(\tau)}{p-1}\ne 0$$

Let us check whether the solution of Eq. (20) has a finite limit w_0 or not for $\tau \to +\infty$. Let $v(\tau) = (L_1 w)^{p-1}$. Then for the derivative function we have:

$$v' = -\left(k_1\phi_1(\tau) + \frac{1-p}{m(p-1)-1} \right) v - k_2 L_2 w + k_3\phi_1(\tau) w + k_4\phi_2(\tau) w^{\beta}$$

We introduce an auxiliary function for analyzing solutions of the last equation:

$$\theta(\tau,\mu) = -\left(k_1\phi_1(\tau) + \frac{1-p}{m(p-1)-1} \right) \mu - k_2 L_2 w + k_3\phi_1(\tau) w + k_4\phi_2(\tau) w^{\beta} \quad (21)$$

where $\mu \in R$. From here it is easy to see that for each value μ of the function $\theta(\tau,\mu)$, the sign is preserved on a certain interval $[\tau_1, +\infty) \subset [\tau_0, +\infty)$ and for all $\tau \in [\tau_1, +\infty)$ one of the inequalities $v'(\tau) > 0$, $v'(\tau) < 0$ is fulfilled. Then, analyzing (21) using Bol's theorem [13], we conclude that the function has a limit $v(\tau)$ at $\tau \in [\tau_1, +\infty)$. It is easy to see that

$\lim\limits_{\tau\to+\infty} \phi_1(\tau) \to 0$, $\lim\limits_{\tau\to+\infty} \phi_3(\tau) \to 0$, $\lim\limits_{\tau\to+\infty} \phi_3(\tau) \to 0$ at $\xi \to (a/b)_{-}^{\frac{p-1}{p-n}}$. Then, taking into account the last limit and $w' = 0$ from (20), we obtain the following algebraic equation for w

$$(p-1)\frac{w^{m(p-1)}}{(m(p-1)-1)^{p-1}} - k_2 w = 0$$

From this equation we obtain that $w = 1$, therefore we have $\varphi(\xi) = \bar{\varphi}(\xi)(1 + o(1))$

in case $\beta > 2p-1$, $q < (p-1)(m - mn + 1)/(p-n)$.

Theorem 7. *The solution of problem (1)-(3) has the asymptotics:*

$$u_4(x,t) = Ct^{\alpha}\bar{g}(\xi)(1 + o(1))$$

where $C = \left(\frac{m(p-1)-1}{p-1} b\gamma \right)^{1/(p(m-1)-1)} \frac{m(p-1)-1}{B(p-1)}$, $\bar{g}$ is the above defined function.

Numerical experiments. It has been established that for the numerical solution of nonlinear problems, the choice of a matching initial approximation, which preserves the nonlinear properties, plays a key role. For this reason, a computational experiment was carried out, which is based on the qualitative properties of the solutions established above, in the case of global solvability. For this purpose, equation (2.1.1) was approximated with the second order of accuracy in spatial coordinates and with the first order in time. For numerical simulation, an iterative process is constructed, in the internal iteration steps of which the node values are calculated by the sweep method. Undoubtedly, iterative methods require a good initial approximation that quickly converges to the desired solution and preserves the qualitative properties of the studied nonlinear processes, which is the main difficulty in the numerical analysis of the problem being solved. A good choice of initial values depending on the value of the numerical parameters included in the equation ensures that this difficulty is overcome. Below we present numerical schemes for the one-dimensional case and some results of numerical experiments.

From fig. 1–3 that the filtration process depends on the density of the medium. Numerical experiments show fast convergence of iterations to the exact solution. This is due to the choice of a suitable initial approximation. The number of iterations is not more than 5 for different values of numerical parameters.

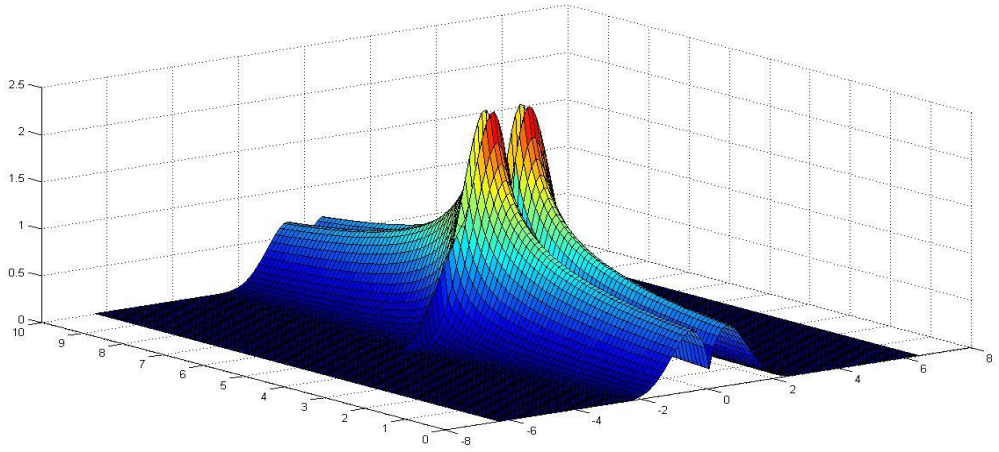


Fig.1. $\beta = 2.5$, p=2.3, n=-1.2, q=1.5, m=1.12

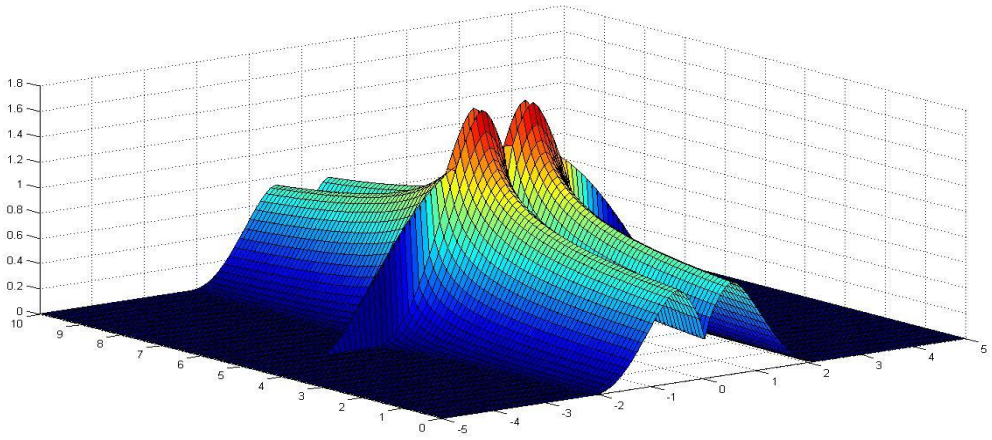


Fig.2. $\beta = 3$, p=2.65, n=-1.2, q=2.5, m=1.25

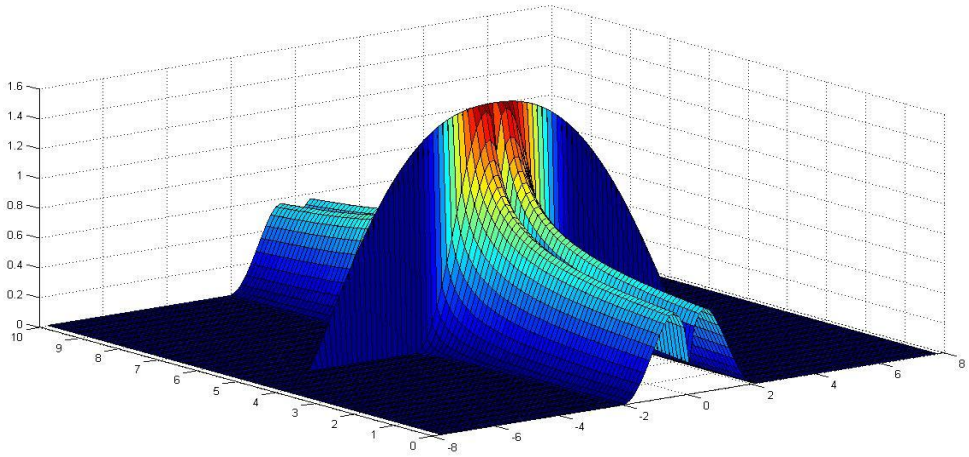


Fig.3. β=2.5, p=2.25, n=-0.75, q=1.95, m=1.65

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