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NUMERICAL SOLUTION OF SUSPENSION FILTRATION THROUGH A RADIAL FILTER UNDER CONSTANT PRESSURE

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Abstract. This study presents a theoretical and numerical investigation of one-dimensional radial suspension filtration through a cylindrical filter medium under constant-pressure conditions. A mathematical model of the filtration process is developed based on the mass conservation equations for the liquid and solid phases coupled with the modified Darcy–Shirato equation. The resulting model is formulated as a nonlinear moving-boundary problem with a free cake–suspension interface, accounting for the dependence of the local permeability and solid volume fraction on the compressive pressure within the cake layer. The numerical results provide the spatiotemporal distributions of the compressive pressure, solid volume fraction, and relative permeability throughout the cake layer.

Keywords: cake layer, cake thickness, constant-pressure filtration, permeability, the cake solidosity, suspension filtration.

Suspenziyani doimiy bosim ostida radial filtr orqali filtrlash jarayonining sonli yechimi

Annotatsiya. Ushbu ishda o‘zgarmas bosim sharoitida radial filtr orqali suspenziyaning suzilishi jarayoni nazariy va sonli tadqiq etilgan. Suyuq va qattiq fazalar uchun massaning saqlanish tenglamalari hamda modifikatsiyalangan Darsi–Shirato tenglamasi asosida suzilish jarayonining matematik modeli qaralgan. Mazkur model, cho‘kmada siqilish bosimiga bog‘liq bo‘lgan lokal o‘tkazuvchanlik va qattiq fazaning hajmiy ulushini hisobga olgan holda, «cho‘kma–suspenziya» chegarasi ega bo‘lgan chiziqli bo‘lmagan harakatlanuvchi chegaraviy masala sifatida shakllantirilgan. Sonli yechimlar cho‘kma qatlami bo‘ylab siqilish bosimi, qattiq fazaning hajmiy ulushi va nisbiy o‘tkazuvchanlikning fazoviy-vaqtiy taqsimotini aniqlash imkonini beradi.

Kalit so‘zlar: keyk-qatlam, keyk-qatlam qalinligi, o‘zgarmas bosimdagi filtrlash, o‘tkazuvchanlik, qattiq fazaning hajmiy ulushi, suspenziyaning suzilishi.

Численное решение задачи фильтрования суспензии через радиальный фильтр при постоянном давлении

Аннотация. Данная работа посвящена теоретическому и численному исследованию одномерного радиального фильтрования суспензии через цилиндрическую фильтровальную перегородку в условиях постоянного давления. На основе уравнений сохранения массы для жидкой и твердой фаз совместно с модифицированным уравнением Дарси–Ширато разработана математическая модель процесса фильтрования. Полученная модель сформулирована как

нелинейная задача с подвижной границей и свободной поверхностью раздела «осадок–суспензия» с учетом зависимости локальной проницаемости и объемной доли твердой фазы от компрессионного давления в слое осадка. Численные результаты позволяют получить пространственно-временные распределения компрессионного давления, объемной доли твердой фазы и относительной проницаемости по всей толщине слоя осадка.

Ключевые слова: кейк-слой, объемная доля твердой фазы, проницаемость, толщина осадка, фильтрование при постоянном давлении, фильтрование суспензии.

Introduction. Technologies for separating suspensions into liquid and solid phases through filtration are fundamental processes across many industries, including oil and gas, chemical, pharmaceutical, agricultural, and environmental. Despite the apparent simplicity of this process, the design of filtration equipment in practice still relies on experimental and empirical data. Consequently, improving the mathematical models of these processes remains a highly relevant task today.

In the classical theory of cake filtration, it is assumed that the structural framework of the cake is formed by a system of point contacts between solid particles. Within this approach, the liquid-phase mass balance equation, together with Darcy's law, provides the mathematical basis for deriving the fundamental differential equation of filtration.

To close the system of equations, constitutive relationships are used, establishing a connection between the main characteristics of the cake (porosity, permeability, compressibility coefficient, etc.) and the effective pressure in the solid skeleton. As a result, the original system is reduced to a single nonlinear differential equation with respect to one independent variable, which, depending on the model adopted, is either the porosity of the cake or the pressure in the solid skeleton.

The resulting equation is solved by analytical or numerical methods, taking into account the initial condition as well as the corresponding boundary conditions at the "filter septum–cake" and "cake–suspension" interfaces. The specific mathematical model of the filtration process is determined by the choice of constitutive relations, as well as by the formulation of the initial and boundary conditions. It is precisely these factors that account for the differences between existing cake filtration models and determine their range of applicability.

A detailed analysis of the scientific literature devoted to filtration and mechanical dewatering of sediments/cakes is presented in the review works of R. Wakeman [1], M. Shirato et al. [2], D. J. Lee and C. H. Wang [3], A. Yükseler et al. [4], and J. Olivier, J. Vaxelaire and E. Vorobiev [5]. These publications systematize current theoretical approaches, experimental research methods, and mathematical models describing the processes of filtration and subsequent mechanical dewatering of cakes.

Although many literature sources link the development of traditional cake filtration theory to the works of Ruth [6,7], subsequent scientists such as Grace, Tiller, Corapcioglu, Shirato, Okamura, Tosun, Wakeman, Stamatakis, Tien, Smiles, Vorobiev, and R. Bürger have also made significant contributions to the formation of this traditional theory.

In [8], the mathematical foundations and primary equations of filtration processes operating under constant-rate filtration and constant-pressure filtration regimes are presented. It is shown that at constant filtration pressure and cake porosity values, the hydraulic resistance of the formed cake is proportional to its quantity (mass per unit area of the filtration surface).

In [9–11], Ruth's theory received experimental validation based on the analysis of the relationship between filtrate volume vs. time curves using compression-permeability (C–P) cells, which allow for the determination of the compression and filtration characteristics of the cake.

In [12], the relationship between the hydrodynamic frictional drag force acting on the liquid phase within the cake layer and the compressive pressure of the solid skeleton is presented as a system of differential equations. Based on this model, the variation in cake porosity and its specific hydraulic resistance across the layer thickness was investigated. Furthermore, mathematical solutions were proposed for filtration processes operating under constant pressure, constant filtration rate, variable pressure and rate, as well as stepwise changes in filtration pressure.

In [13], the mathematical modeling of the dynamics of pressure filtration processes for flocculated suspensions was investigated in detail. A mathematical model was developed based on

non-linear partial differential equations of a mixed hyperbolic-parabolic type, describing the spatial-temporal evolution of the solid phase volume fraction during the filtration process. In [14], phenomenological models of sedimentation–consolidation processes of flocculated suspensions, treated as a mixture of two interacting continuous media, were comprehensively studied. It was shown that in a one-dimensional formulation, the mathematical model reduces to a strongly degenerate non-linear partial differential equation of a mixed hyperbolic-parabolic type, describing the evolution of the sedimentation and consolidation process of the dispersed system.

In [15], a simplified mathematical model of cake filtration was proposed, taking into account the effect of hydraulic pressure redistribution at the interface between the filter medium and the cake layer. This model allows for the evaluation of the influence of boundary hydrodynamic effects on the characteristics of the filtration process. In [16], a unified non-linear mathematical model was developed to predict the sequential progression of filtration and mechanical expression processes under a constant pressure drop. The model is based on a modified system of Darcy–Terzaghi equations, in which the non-linear dependence of the specific hydraulic resistance of the cake on pressure is approximated by a power function, and Darcy's law is written in linear form. A significant feature of the model is that it accounts for the spatial heterogeneity of the structure and the compressibility of the forming filter cake, providing a more accurate description of its formation, consolidation, and mechanical dewatering compared to traditional models.

In [17,18], the theory of filter cake was further developed to incorporate not only its reversible elastic properties but also the irreversible plastic deformations occurring during the loading process. The authors developed a system of dynamic equations describing the behavior of the cake under various mechanical loading regimes, including initial loading during filtration, unloading after the removal of external pressure, and subsequent reloading. Based on the proposed model, a quantitative assessment was performed regarding the influence of the plastic properties of the cake on its compaction kinetics, changes in layer thickness, and the integral characteristics of the filtration process. In the literature on filtration, processes are classified into several types or regimes: complete blocking, intermediate blocking, standard blocking, and cake filtration (i.e., filtration with cake formation). Each of these regimes is characterized by specific hydrodynamic flow properties, for the description of which corresponding mathematical models have been developed [19–22]. The choice of the model is determined by the type of filtration scenario: filtration under a constant pressure drop or at a constant flow rate. In the first case, the modeling aims to establish the relationships between the instantaneous filtration rate $q_{\lambda s}(t)$ at time t and the cumulative filtrate volume $Q(t)$, where $Q(t)$ is defined as the integral of $q_{\lambda s}(t)$ over time ($Q=\int_0^t q_{\lambda s}(t)dt$). In the second case, priority is given to analyzing the relationship between the pressure required to maintain a constant flow rate and the volume of processed filtrate $Q(t)$, which is a key factor for evaluating the efficiency of the filtration process.

Filtration scheme. The process of separating a suspension into liquid and solid phases via cylindrical filtration with the formation of a cake layer is considered. The mathematical model is based on the geometry of a cylindrical filter medium with a constant radius r_m, through which the filtrate is discharged. Due to the retention of solid particles on the surface of the filter medium, the continuous formation and growth of a porous cake layer occurs, which exerts additional hydraulic resistance to the filtration flow.

At each moment in time t, the cake occupies the region $r_m \le r \le R(t)$, where $R(t)$ denotes the current radius of the outer boundary of the cake, which coincides with the "suspension–cake" phase interface. As new solid particles arrive, the outer radius of the cake monotonically increases, as a result of which the conditions $R(t) \ge r_m$ and $\frac{dR}{dt} > 0$ are satisfied. The resulting mathematical formulation therefore constitutes a moving-boundary (Stefan-type) problem, in which the interface position is governed by the kinetics of solid particle deposition and cake growth.

The radial coordinate, r, is measured from the cylinder axis outward, extending from the surface of the filter medium toward the bulk suspension, and is defined as positive in this direction. The liquid filtration flow, however, is directed toward the filter medium, i.e., opposite to the positive radial direction, corresponding to decreasing values of the radial coordinate. Consequently, the radial

component of the liquid-phase velocity, q_λ, is negative. During cake consolidation, the solid skeleton undergoes a gradual inward displacement toward the filter medium as a result of compression induced by the hydrodynamic pressure of the flowing liquid. Accordingly, the radial velocity of the solid phase, q_s, is also negative in the adopted coordinate system.

The region $r > R(t)$ is occupied by the feed suspension with a prescribed solid particle concentration and serves as the feed domain of the filtration process. At the moving interface, $r = R(t)$, suspended particles are continuously incorporated into the growing cake layer. Accordingly, the interface is governed by mass conservation conditions for both the liquid and solid phases, together with a kinematic condition that determines the velocity of the moving boundary. The combined application of these interfacial conditions ensures a physically consistent description of cake growth and the evolution of its internal structure during cylindrical filtration.

The mathematical description of cake filtration requires establishing the relationships among the principal process variables, including the cumulative filtrate volume per unit filtration area, cake thickness, average cake porosity, and the applied pressure. To this end, the model is formulated on the basis of the following governing relations: (i) the Darcy law describing the filtration kinetics; (ii) the mass conservation equations for the liquid and solid phases; (iii) constitutive relationships linking the hydrodynamic drag resistance to the structural properties of the cake; (iv) the force balance governing the relationship between the liquid-phase pressure and the total hydrodynamic drag exerted on the solid particles; and (v) the boundary conditions prescribed at the filter medium–cake interface and at the moving cake–suspension interface.

The governing equations for fluid flow in porous media are traditionally formulated on the basis of Darcy's law. However, in compressible filter cakes formed by the deposition of suspended solid particles, the relative motion of the solid skeleton with respect to the liquid phase must also be taken into account. Under these conditions, the classical Darcy equation is generalized by the Darcy–Shirato formulation, which incorporates the motion of both phases. Accordingly, the Darcy–Shirato equation reported in [23] can be expressed in cylindrical (radial) coordinates as follows:

$$\frac{\partial p_\lambda}{\partial r} = -\frac{\mu(1-\varepsilon_s)}{k}\left(\frac{q_\lambda}{1-\varepsilon_s} - \frac{q_s}{\varepsilon_s}\right), \tag{1}$$

where p_λ is the pressure in the liquid phase, k is the permeability, μ is the viscosity of the suspension, ε_s is the cake solidosity (or particle volume fraction of cake), t is time, r is the radial coordinate, q_λ is the filtration rate of the liquid phase, q_s is the filtration rate of the solid phase.

As the fluid flows through the porous cake, the suspended solid particles experience hydrodynamic drag forces generated by viscous interaction with the liquid phase. Because the particles are in direct mechanical contact, the local drag forces acting on individual particles are transmitted through the interconnected contact network of the cake and accumulate along the direction of filtration. As a result, compressive stresses develop within the solid skeleton of the cake [24–27].

The progressive increase in compressive stress causes the cake structure to consolidate, bringing neighboring particles closer together and thereby reducing the local porosity. Consequently, the cake becomes increasingly compact toward the filter medium, where the compressive stress reaches its maximum value. The highest degree of consolidation therefore occurs in the vicinity of the filter medium, while the outer regions of the cake remain comparatively more porous.

For one-dimensional cake filtration, the isotropic stress components represented by the liquid-phase pressure, p_λ, and the compressive pressure, p_s, constitute the dominant components of the stress tensor. The consolidation behavior of the cake is commonly described using constitutive relationships that couple the liquid-phase pressure with the compressive stress developed within the solid matrix. Several of the simplest constitutive models proposed in the literature can be expressed in the following form [24]:

$$\frac{\partial p_\lambda}{\partial r} + \frac{\partial p_s}{\partial r} = 0. \tag{2}$$

At the filter medium/cake interface, the velocity of the solid phase is assumed to be zero, i.e., $q_s\big|_{r=r_m} = 0$. Under this boundary condition, the total filtration velocity is equal to the liquid-phase filtration velocity

$$q_\lambda\big|_{x=0} = q_{\lambda m} = -\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\bigg|_{r=r_m}. \quad (3)$$

At any point within the cake layer at each moment in time, the total filtration rate remains constant. This means that the sum of the liquid phase filtration rate and the velocity of the solid phase movement through of the cake remains unchanged and is equal to the overall filtration rate:

$$q_\lambda + q_s = q_{\lambda m}. \quad (4)$$

where is $q_{\lambda m}$ the total filtration rate.

Taking into account equations (3) and (4), the right - hand side of equation (1) can be rewritten as follows:

$$\frac{q_\lambda}{1-\varepsilon_s} - \frac{q_s}{\varepsilon_s} = \frac{\varepsilon_s q_\lambda - (1-\varepsilon_s)(q_{\lambda m} - q_\lambda)}{\varepsilon_s(1-\varepsilon_s)} = \frac{\varepsilon_s q_\lambda - (1-\varepsilon_s)q_{\lambda m} + (1-\varepsilon_s)q_\lambda}{\varepsilon_s(1-\varepsilon_s)} = \frac{q_\lambda - (1-\varepsilon_s)q_{\lambda m}}{\varepsilon_s(1-\varepsilon_s)}. \quad (5)$$

Taking into account (5), we rewrite (1) as follows:

$$\frac{\partial p_\lambda}{\partial r} = -\frac{\mu(1-\varepsilon_s)}{k}\left(\frac{q_\lambda - (1-\varepsilon_s)q_{\lambda m}}{\varepsilon_s(1-\varepsilon_s)}\right).$$

As a result, we arrive at the conclusion that the velocity of the liquid phase is equal to:

$$q_\lambda = (1-\varepsilon_s)q_{\lambda m} - \varepsilon_s\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}. \quad (6)$$

Taking into account (2) and (3), from (6) we obtain

$$q_\lambda = -(1-\varepsilon_s)\left[\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\right]\bigg|_{r=r_m} - \varepsilon_s\left[\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\right]. \quad (7)$$

To obtain the filtration equation, we use the balance equation

$$\frac{\partial \varepsilon_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(rq_\lambda). \quad (8)$$

Substituting (5) into (6), we have

$$-\frac{\partial \varepsilon_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left(\varepsilon_s\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r} + (1-\varepsilon_s)\left[\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\right]\bigg|_{r=r_m}\right)\right\},$$

from which

$$\frac{\partial \varepsilon_s}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left\{r\varepsilon_s\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\right\} - \frac{1}{r}\left[\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\right]\bigg|_{r=r_m}\frac{\partial}{\partial r}(r(1-\varepsilon_s)). \quad (9)$$

Equation (9) constitutes the governing equation of the cake filtration model. Equation (9) is solved subject to the corresponding initial and boundary conditions, including those prescribed at the moving cake–suspension interface, $R(t)$. Because the interface location is part of the solution, the problem belongs to the class of Stefan-type moving-boundary problems, and the evolution of $R(t)$ is determined from an additional interface (kinematic) equation.

Accordingly, the equation governing the growth of the cake layer is given by

$$\frac{\partial \varepsilon_s}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\varepsilon_s\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(r(1-\varepsilon_s)\left[\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\right]_{r=r_m}\right). \quad (10)$$

If the filtration process starts with a clean filter, i.e., without prior permeation of the liquid, the following initial condition may be assumed:

$$R(0) = 0. \quad (11)$$

Under constant-pressure filtration with an applied pressure p_0, the governing equations are subject to the following boundary conditions:

$$p_\lambda = p_0,\ p_s = 0,\ \varepsilon_s = \varepsilon_s^0 \text{ at } r = R(t), \quad (12a)$$

$$-\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r} = \frac{-p_\lambda}{R_m\mu} \text{ at } r = r_m. \quad (12б)$$

In the constant-pressure filtration regime, the filtrate flow rate at the filter outlet,

$q_{\lambda m} = -\left[\frac{k}{\mu}\frac{\partial p_\lambda}{\partial r}\right]_{r=r_m}$, is calculated as a time-dependent function from the solution of the governing equations.

The geometry of the pore space in a filter cake is determined by the shape, size, packing nature, and relative arrangement of its constituent particles. During the process of cake formation via the deposition of solid particles, as well as under the action of hydrodynamic forces caused by the filtration flow, its internal structure continuously changes. In this regard, in many mathematical models, the cake solidosity (or particle volume fraction of cake) is represented as the product of its initial unstressed value (ε_s^0) and a power function of the compressive pressure (p_s).

$$\varepsilon_s = \varepsilon_s^0\left(1+\frac{p_s}{p_A}\right)^{\beta}, \tag{13a}$$

where p_A is the characteristic pressure, β is the exponent – constant value.

The permeability of the cake, which characterizes its ability to transmit fluid, is determined experimentally from the relationship between the filtration velocity and the applied pressure drop and is described by the following constitutive equation [25–27]:

$$k = k^0\left(1+\frac{p_s}{p_A}\right)^{-\delta}, \tag{13б}$$

where k^0 is the value of k at $p_s = 0$, δ is the exponent – constant value.

To solve this system of equations, we use relations (2) and (13) to transform equations (9) and (10), expressing them in terms of p_s

$$\frac{\partial p_s}{\partial t} = -\frac{p_A k^0}{\mu\cdot\beta}\left(1+\frac{p_s}{p_A}\right)^{1-\beta}\frac{1}{r}\frac{\partial}{\partial r}\left(r\left(1+\frac{p_s}{p_A}\right)^{\beta-\delta}\frac{\partial p_s}{\partial r}\right) + q_{\lambda m}\frac{\partial p_s}{\partial r} - \frac{p_A}{\varepsilon_s^0\beta}\frac{q_{\lambda m}}{r}\left(\left(1+\frac{p_s}{p_A}\right)^{1-\beta} - \varepsilon_s^0\left(1+\frac{p_s}{p_A}\right)\right), \tag{14}$$

$$\frac{dR}{dt} = \frac{\varepsilon_s^0}{\varepsilon_s^0-\varepsilon_{s_0}}\left[\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)^{-\delta}\frac{\partial p_s}{\partial r}\right]_{r=R^-} - q_{\lambda m}. \tag{15}$$

where

$$q_{\lambda m} = \left[\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)^{-\delta}\frac{\partial p_s}{\partial r}\right]_{r=r_m}.$$

The initial and boundary conditions for p_s have the following form:

$$R(t) = 0 \text{ at } t = 0, \tag{16}$$

$$p_s = 0 \text{ at } r = R(t), \tag{17a}$$

$$\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)^{-\delta}\frac{\partial p_s}{\partial r} = \frac{p_0 - p_s}{R_m\mu} \text{ at } r = r_m. \tag{17б}$$

These systems of equations belong to the class of free (moving) boundary problems, which are also known as Stefan problems. The finite difference method is applied for their numerical solution [28–29].

Numerical results. Numerical results for (14) - (17) were obtained for the following values of the initial parameters [20]: $p_A = 10^3$ Pa, $\mu = 10^{-3}$ Pa·s, $k^0 = 1\cdot10^{-13}$ m^2, $\varepsilon_s^0 = 0{,}2$, $\varepsilon_{s_0} = 0{,}0076$, $\beta = 0{,}13$, $\delta = 0{,}57$.

Some of the results are shown in Figs. 1–3. As follows from the results presented in Fig. 1, the increase in cake thickness exhibits a pronounced nonlinear character. This behavior is due to the gradual formation of the compressive pressure distribution throughout the cake thickness, accompanied by compaction of its porous structure and a corresponding increase in the hydraulic resistance of the filtration layer. As a result, during filtration under a constant pressure drop regime, the filtration flow rate decreases monotonically with time, which, in turn, leads to a decrease in the

rate of cake thickness growth.

Figure 2a shows the spatial distributions of the compressive pressure within the cake layer for several fixed time instants. In accordance with the adopted boundary conditions, the maximum value of the compressive pressure is attained at the surface of the filter medium ($r = r_m$), after which it decreases monotonically across the cake thickness, vanishing at the moving "cake–suspension" interface. As the filtration process progresses, an increase in compressive pressure is observed at the cake layer, indicating progressive consolidation and compaction of the cake structure.

Figure 2b illustrates the spatial distribution of the cake solidosity within the cake layer at several representative time instants. The nature of the changes in these profiles is directly determined by the distribution of the compression pressure shown in Fig. 2a. The maximum value of the cake solidosity, corresponding to the most compact structure of the cake, is observed near the surface of the filter medium, where the effective stress in the cake skeleton reaches its maximum. With increasing distance from the filter medium, the value of ε_s monotonically decreases and reaches its minimum value at the moving "cake–suspension" interface, which coincides with the initial volume concentration of the solid phase in the feed suspension. Over time, an increase in the cake solidosity is observed throughout the entire thickness of the layer, indicating the progressive compaction and consolidation of the filter cake under the action of the continuous hydrodynamic pressure of the filtration flow.

Figure 2c presents the spatial distributions of the relative permeability across the cake thickness for fixed time instants: $t = 450$ s, $t = 900$ s, and $t = 1800$ s. In accordance with the power-law dependence adopted in the model (Eq. (13b)), the relative permeability increases monotonically from the surface of the filter medium toward the moving "cake–suspension" interface. The minimum permeability values are observed near the filter medium, where the cake structure is most compacted due to the maximum effective pressure. As the outer boundary of the layer is approached, the compressive pressure decreases and the permeability increases, reaching its initial value at the "cake–suspension" interface, where no effective stresses are present in the solid skeleton. Over time, the relative permeability decreases at all cross-sections of the cake, reflecting the progressive compaction of its porous structure and the resulting increase in the overall hydraulic resistance of the filtration layer.

Fig. 3 shows the time dependencies of the compression pressure, the cake solidosity, and the relative permeability, calculated for a fixed point in the cake. As the filtration process develops, a monotonic increase in the effective compression pressure is observed. This growth in effective pressure is accompanied by intense compaction of the cake structure, which is clearly manifested in a continuous increase in the cake solidosity. Concurrently, a monotonic decrease in relative permeability is recorded, which is caused by the reduction in the pore space volume and the decline in the hydraulic conductivity of the filtration layer.

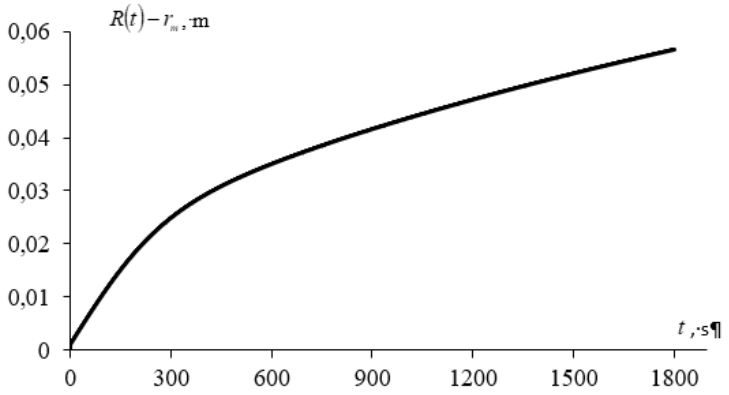

Figure. 1. Dynamics of sediment layer thickness on the filter surface.

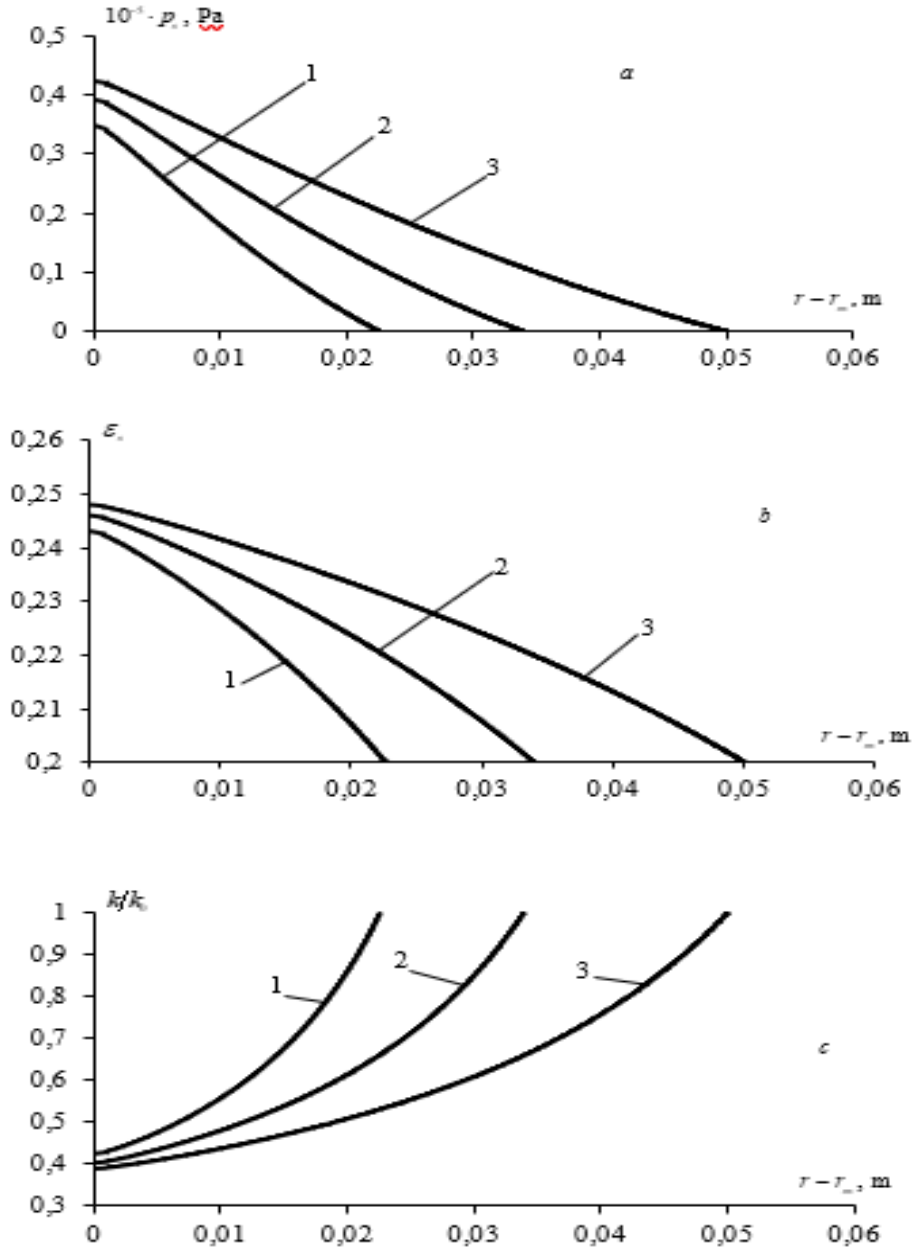

Figure. 2. Distribution of p_λ (a), ε_s (b), k/k_0 (c) by sediment thickness at $t = 450$ (1); 900 (2); 1800 (3) s.

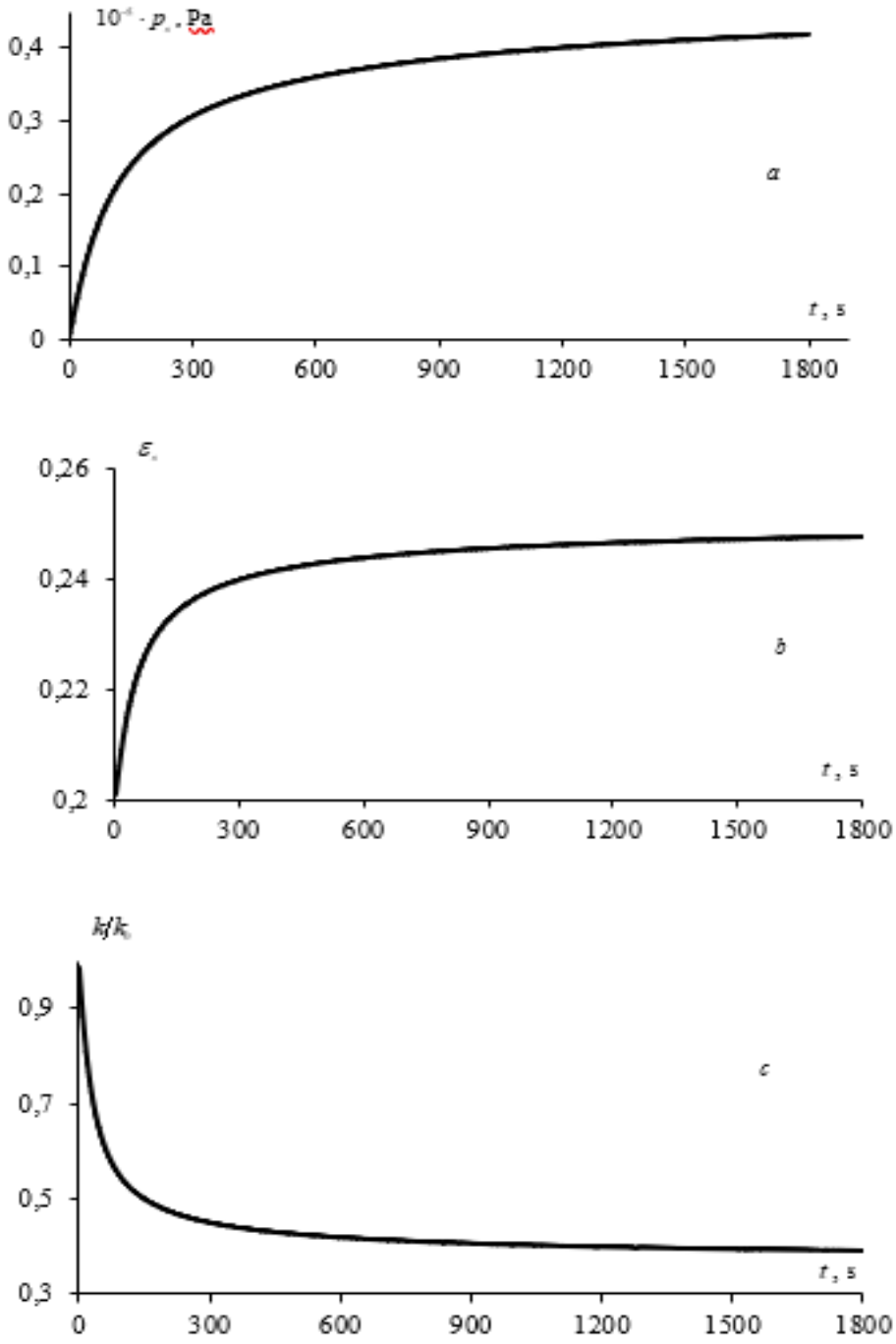

Figure. 3. Dynamics of p_λ (a), ε_s (b), k/k_0 (c) at point $x = 0{,}002$ m at $t = 1800$ c.

Conclusion. In this work, a comprehensive theoretical and numerical study of the suspension filtration process through a cylindrical filter medium under a constant pressure regime with the formation of a compressible cake layer was carried out. The fundamental filtration equation was derived based on the continuity equations for the liquid and solid phases in conjunction with the modified Darcy–Shirato law. Taking into account the empirical dependencies of the cake solidosity and permeability on the compression pressure, the resulting system of nonlinear partial differential equations was formulated relative to the compression pressure in the solid of the cake. To numerically solve the obtained Stefan-type boundary value problem with a moving "cake–suspension" interface, the finite difference method was successfully applied in combination with a front-tracking (front-catching) scheme onto the spatial grid nodes.

Based on the developed model, the spatiotemporal dynamics of the main characteristics of the process were investigated in detail, which allowed for the formulation of the following key conclusions: It was established that the growth of the cake thickness over time has a pronounced nonlinear character. An analysis of the distribution of internal characteristics across the cake thickness showed that the maximum values of the compression pressure and the cake solidosity are achieved directly at the surface of the filter medium. Toward the moving outer boundary, the effective pressure monotonically decreases to zero, and the solids content drops to its minimum value, which is equal to the initial concentration of the suspension. Conversely, the relative permeability increases from its minimum value at the substrate to unity at the boundary with the suspension.

The results obtained indicate that the proposed axisymmetric mathematical model correctly reproduces the main physical patterns of cylindrical filtration with the formation of a compressible cake, including the development of the effective stress distribution, consolidation of the porous structure, changes in filtration properties, and the continuous movement of the "cake–suspension" interface. The developed approach can serve as a foundation for the further modeling and optimization of filtration processes in cylindrical filtration equipment under various hydrodynamic regimes and rheological properties of disperse systems.

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