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THEORETICAL FOUNDATIONS AND COMPUTER IMPLEMENTATION OF RATIONAL PARAMETERIZATION OF ALGEBRAIC MANIFOLDS IN THE MAPLE SYSTEM

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Abstract. We considered the main theoretical aspects of the parameterization of algebraic manifolds. Presented calculations of parameterization, both in a computer algebra system and manually. For clarity, we also implemented calculations in the Maple computer algebra system using the *algcurves* package. Based on the results, a detailed algorithm for calculating the parameterization of algebraic curves was also developed. Thus, the theoretical research was completed with practical implementation, confirming the correctness of the approach.

Keywords: Algebraic manifolds, computer algebra, genus of a curve, Maple, rational parameterization, singular points.

Maple tizimida algebraik ko’pxilliklarni ratsional parametrlashining nazariy asoslari va kompyuterda amalga oshirilishi

Annotatsiya. Algebraik ko’pxilliklarni parametrizatsiyalash asosiy nazariy jihatlarini ko'rib chiqildi. Parametrizatsiyalash hisob-kitoblarini ham kompyuter algebra tizimida, ham qo'lda taqdim etdik. Aniqlik uchun, shuningdek, *algcurves* paketidan foydalangan holda Maple kompyuter algebra tizimida hisob-kitoblarni amalga oshirdik. Natijalar asosida algebraik ko’pxilliklarni parametrizatsiyasini hisoblash uchun batafsil algoritm ham ishlab chiqildi. Shunday qilib, nazariy tadqiqot amaliy tatbiq bilan yakunlandi va yondashuvning to'g'riligini tasdiqladi.

Kalit so‘zlar: Algebraik ko’pxilliklar, egri chiziq jinsi, kompyuter algebra, Maple, ratsional parametrizatsiya, singular nuqtalar.

Теоретические основы и компьютерная реализация рациональной параметризации алгебраических многообразий в системе Maple

Аннотация. Рассмотрено основные теоретические моменты параметризации алгебраических многообразий. Были проведены вычисления параметризации, как и в системе компьютерной алгебры так и в вручную. Также для наглядного представления реализовали вычисления в системе компьютерной алгебры Maple с использованием пакета *algcurves*. Основываясь на результаты, также составлен подробный алгоритм вычисления параметризации алгебраических кривых. Таким образом, теоретическое исследование завершено практической реализацией, подтвердивший корректность подхода.

Ключевые слова: Алгебраические многообразия, компьютерная алгебра, Maple, особые точки, рациональная параметризация, род кривой.

1. Introduction.

1.1 Objective of the work.

The main objective of this work is to substantiate the concepts of parameterizability of algebraic manifolds and implement the corresponding algorithms in the Maple system using the *algcurves* package.

1.2 Relevance of the topic.

Currently, one of the rapidly developing branches of mathematics is algebraic geometry, especially issues related to the parametric description of algebraic manifolds are becoming increasingly important. Sometimes, when plotting graphs in the Maple system, inaccuracies may occur, for example, related to the smoothing of certain parts of the graph. It is in such cases that using parameterization can avoid such distortions and obtain a more accurate representation. In this work, we considered the possibility of implementing parameterization using the *algcurves* package in the Maple system, which makes the work relevant both from the point of view of theory and its practical application.

2. Theoretical part.

2.1 Algebraic manifolds: definitions and examples.

Let K be some field, and $f_1, f_2, \dots, f_s$ be polynomials in $K[x_1, x_2, \dots, x_n]$. Let

$$V(f_1, f_2, \dots, f_s) = \{(a_1, a_2, \dots, a_n) \in K^n : f_i(a_1, a_2, \dots, a_n) = 0 \text{ for all } 1 \le i \le s\} .$$

$V(f_1, f_2, \dots, f_s)$ is called the affine manifold defined by the polynomials $f_1, f_2, \dots, f_s$. [1]

Simply put, an affine manifold $V(f_1, f_2, \dots, f_s) \in K$ is the set of solutions to the system of equations $f_1(x_1, x_2, \dots, x_n) = \dots = f_s(x_1, x_2, \dots, x_n) = 0$

2.2 Rational Parameterization.

To begin, consider a system of equations over $\mathbb{R}$

$$\begin{cases} 2x + 3y - z = 2 \\ x + 2y + z = 1 \end{cases}$$

It is known that system (1) has infinitely many solutions. To describe the solutions, we use the Gaussian elimination method. We transform system (1) into the form

$$\begin{cases} y + 3z = 0 \\ x - 5z = 1 \end{cases}$$

Let $z = t$, where t is an arbitrary real number, then the solution of (1) can be written as

$$\begin{cases} x = 1 + 5t \\ y = -3t \\ z = t \end{cases}$$

Where t is called a parameter, and (2) is a parameterization of the solution set of system (1).

In a more specific case, sometimes the coordinates of a parameterization can be expressed rationally. That is, each coordinate is represented as a rational expression. This type is called a rational parameterization, and it allows us to describe the set of points on a manifold through parameters, where different values of the parameters yield all (or almost all) points of the given set.

One criterion for whether a curve can be rationally parameterized is its genus. More precisely, if the genus of the curve is 0, then the curve admits a rational parameterization. The genus of a curve can be computed in the Maple system using the genus function. If the result is 0, constructing a rational parameterization can be done using the parametrization function from the algcurves package.

2.3 Singular Points.

According to [2], singular points of a curve are points:

1. in which both derivatives simultaneously vanish: $x'(t) = y'(t) = 0$

2. or they are isolated points where the imaginary parts of its complex branches vanish.

Singular points of a curve can be computed using the function $singularities(F, x, y)$.

3. Methodological Approaches to Parameterization.

3.1 Parameterization Tasks.

Let's look at an example of how to construct an implicit representation. In our case, consider a curve in the plane:

$$\begin{cases} x = t + 2 \\ y = t^2 \end{cases}$$

Since it is not yet known whether this curve lies on any affine manifold, we proceed to construct an equation by expressing t from the first equation of system (3) as $t = x - 2$ and substituting t into the second, obtaining $y = (x-2)^2$. It follows that system (3) describes the affine manifold $V(y - x^2 + 4x - 4)$.

3.2 Parameterization Algorithms.

Let's consider algorithms for computing parameterization in the computer algebra system Maple.

Step 1. Determine the difference between deg and $ldeg$ for the chosen f. In this work, we will limit ourselves to the most common results, i.e., when the difference is 1 or 2.

Step 2. Compute the singular points and the genus of the curve using the singularities and genus commands, respectively (further computations become meaningful only when the genus of the curve is 0).

- Consider the case where the difference between deg and $ldeg$ is 1:

$$f(x,y) = f_a(x,y) + f_{a-1}(x,y) \quad (4)$$

Step 3. Perform the substitution $x = xz, y = yz$ and substitute into the curve equation.

Step 4. Divide by $ldegz$ and compute the coefficient of z for the corresponding powers.

Step 5. Compute the values $f_a(1,t)$ and $f_{a-1}(1,t)$.

Step 6. Compute the parameterization using the formula

$$x = \frac{-f_{a-1}(1,t)}{f_a(1,t)},\ y = \frac{-tf_{a-1}(1,t)}{f_a(1,t)}. \quad [3]$$

Step 7. Substituting the computed parameterization into (4) yields the identity 0, which demonstrates the correctness of our parameterization.

- In the case where the difference between deg and $ldeg$ is 2, the process begins with the first two steps indicated above:

$$f(x,y) = f_a(x,y) + f_{a-1}(x,y) + f_{a-2}(x,y). \quad (5)$$

Step 8. Perform the substitution $x = xz, y = yz$ and substitute into the curve equation.

Step 9. Divide by and make the substitution $x = 1, y = \tau$.

Lemma. Let $f = f_a + f_{a-1} + f_{a-2}$. $V(f)$ is rationally parameterizable if and only if we can parameterize $V(y^2 - D(x))$, where $D(\tau) = [f_{a-1}(1,\tau)]^2 - 4f_a(1,\tau)f_{a-2}(1,\tau)$. Alternatively, set $D = P^2Q$.

Step 10. Compute the values D, P, Q and if Q is expressed as

$$(\tau) = \alpha + \beta\tau + \gamma\tau^2, \quad \gamma \neq 0 \quad \text{then} \quad \tau_0 = \begin{cases} \frac{\beta}{\tau^2 - \gamma}, & \text{if } \alpha = 0 \\ \frac{4\alpha\tau}{(t-\beta)^2 - 4\alpha\gamma} & \text{otherwise} \end{cases} \quad [3]$$

Step 11. **Having computed the value τ, find P and R in terms of the parameter t.**

Step 12. **Obtain the parameterization using the formula**

$$x = \frac{-f_{a-1}(1,\tau_0)+P(\tau_0)R(\tau_0)}{2f_a(1,\tau_0)}, \quad y = \tau_0 x. \qquad [3]$$

Step 13. **Substituting the computed parameterization into (5) yields the identity 0, which demonstrates the correctness of the parameterization.**

1. **Implementation of Examples in the Maple System**

Example 1. Find the parameterisation for a given curve.

Listing 1. Maple

Definition of polynomial and variables

$$f := y^2 - x^3 - x^2;$$

$$vars := [x, y];$$

Searching for special points on a projective plane

$$singularities(f, x, y);$$

$$f := -x^3 - x^2 + y^2$$

$$\{\{[0, 0, 1], 2, 1, 2\}\}$$

Calculating the genus of a curve

$$genus(f, x, y);$$

$$0$$

Transition to homogeneous coordinates (z) and component extraction

$$f1 := collect(normal(subs([x = x * z, y = y * z], f) / z\text{^}2), z);$$

$$f1 := -x^3 z - x^2 + y^2$$

Coefficient at the first degree z

$$f3 := coeff(f1, z, 1);$$

$$f3 := -x^3$$

Coefficient at zero degree z (free term)

$$f2 := coeff(f1, z, 0);$$

$$f2 := -x^2 + y^2$$

Parameterisation of a curve in the neighbourhood of an infinitely distant point

$$f[d] := factor(eval(f3, [x = 1, y = t]));$$

$$f[d-1] := factor(eval(f2, [x = 1, y = t]));$$

$$f_d := -1$$

$$f_{\{d-1\}} := (t - 1)(t + 1)$$

Explicit parameterisation

$$Par_f := [x = -f[d-1]/f[d], y = -t * f[d-1]/f[d]];$$

$$Par_f := [x = (t - 1)(t + 1), y = t(t - 1)(t + 1)]$$

Checking the parameterisation by substituting into the original equation

$$simplify(subs([x = x_param, y = y_param], y^2 - x^3 - x^2));$$

$$0$$

Example 2. Find the parameterisation for a given curve

Listing 2. Maple

$$f := 2 * x\text{^}2 * y\text{^}2 + 2 * y\text{^}4 + 4 * x\text{^}2 * y - 2 * x * y\text{^}2 + 6 * y\text{^}3 + x\text{^}2 - 4 * y * x + 4 * y\text{^}2;$$

$$f := 2x^2y^2 + 2y^4 + 4x^2y - 2xy^2 + 6y^3 + x^2 - 4xy + 4y^2$$

$$vars := [x, y]:$$

$$singularities(f, x, y);$$

$$\{[[0, 0, 1], 2, 1, 1], [[1, 0, 0], 2, 1, 2], [[3/2, -3/2, 1], 2, 1, 2]\}$$

$$genus(f, x, y);$$

$$0$$

$$df := degree(f, [x, y]);$$

$$ldf := ldegree(f, [x, y]);$$

$$df := 4$$

$$ldf := 2$$

$$Subs_z := [x, y] = \sim map(`*`, [x, y], z);$$

$$Subs_z := [x = xz, y = yz]$$

$$fHom := collect(normal(eval(f, Subs_z)/z\hat{}ldf), z, expand);$$

$$fHom := (2x^2y^2 + 2y^4) * z^2 + (4x^2y - 2xy^2 + 6y^3)z + x^2 - 4yx + 4y^2$$

$$fHom := eval(fHom, [x = 1, y = tau]);$$

$$fHom := (2\tau^4 + 2\tau^2)z^2 + (6\tau^3 - 2\tau^2 + 4\tau)z + 1 - 4\tau + 4\tau^2$$

$$D_f := factor(discrim(fHom, z));$$

$$Q_f := op(2, \%);$$

$$solve(P\hat{}2 = D_f/Q_f, P);$$

$$P_f := \%[2];$$

$$D_f := 4(\tau^2 + 2)(\tau + 1)^2\tau^2$$

$$Q_f := \tau^2 + 2$$

$$-2\tau^2 - 2\tau, 2\tau^2 + 2\tau$$

$$P_f := 2\tau^2 + 2\tau$$

$$local\ gamma;$$

$$Q_template := alpha + beta * tau + gamma * tau\hat{}2:$$

$$Subs_(tau_template) := tau = 4 * alpha * t/((t - beta)\hat{}2 - 4 * alpha * gamma);$$

$$Subs_{tau_{template}} := \tau = 4\alpha\frac{t}{(t - \beta)^2 - 4\alpha\gamma}$$

$$[coeffs](Q_f - Q_template, tau);$$

$$solve(\%, [alpha, beta, gamma])[1];$$

$$Subs_tau := eval(Subs_(tau_template), \%);$$

$$[-\gamma + 1, -\beta, -\alpha + 2]$$

$$[\alpha = 2, \beta = 0, \gamma = 1]$$

$$Subs_{tau} := \tau = 8\frac{t}{t^2 - 8}$$

$$P_f := factor(eval(P_f, Subs_tau));$$

$$P_f := 16t\frac{t^2 + 8t - 8}{(t^2 - 8)^2}$$

$$factor(eval(Q_f, Subs_tau));$$

$$solve(R\hat{}2 = \%, R):$$

$$R_f := \%[2];$$

$$2\frac{(t^2 + 8)^2}{(t^2 - 8)^2}$$

$$R_f := -\sqrt{2}\frac{t^2 + 8}{t^2 - 8}$$

$$coeff(fHom, z, 1);$$

$$f1_f := factor(eval(\%, Subs_tau));$$

$$coeff(fHom, z, 2);$$

$$f2_f := factor(eval(\%, Subs_tau));$$

$$6\tau^3 - 2\tau^2 + 4\tau$$

$$f1_f := 32t\frac{t^4 - 4t^3 + 80t^2 + 32t + 64}{(t^2 - 8)^3}$$

$$2\tau^4 + 2\tau^2$$

$$f2_f := 128t^2\frac{t^4 + 48t^2 + 64}{(t^2 - 8)^4}$$

$$x = (-f1_f + P_f * R_f)/(2 * f2_f):$$

$$Par_x := factor(normal(\%)):$$

$$Par_y := y = rhs(Subs_tau) * rhs(Par_x):$$

$$simplify(eval(f, [Par_x, Par_y])); \quad 0$$

$$> \ Par_f := factor(parametrization(f, x, y, t));$$

$$Par_f := \left[-8(t-4) * \frac{(t-2)^2}{(t^2-8t+8)(t^2-4t+8)}, -2\frac{(t-2)^2}{t^2-4t+8}\right]$$

5. Analysis and Discussion of Results.

5.1 Efficiency of the Applied Methods.

In this work, both analytical methods and the computer algebra tools of the Maple system were used to consider the examples. This made it possible not only to automate the computation process but also to gain a deeper understanding of the essence of each step of the algorithm, which was created based on the performed calculations and adapted to the specific types of curves we worked with.

5.2 Prospects for Further Work. This paper considered cases where the difference between deg and $ldeg$ is 1 or 2. Under these conditions, the proposed rational parameterization algorithm is successfully applicable and yields correct results. However, when the difference increases to three or more, these methods require additional changes of variables to ensure the equalization of degrees and the restoration of a rational dependence. It should also be noted that the parameterizations obtained using this algorithm and the parameterization obtained using the parametrization command from the algcurves package may sometimes not coincide. In such cases, conditions can be determined under which the parameters become identical, ensuring the equivalence of both forms of curve representation. In particular, the proposed algorithm can be used to analyze singular points and the structure of algebraic curves, which is an important step in studying the conditions for the existence of resonances in Hamiltonian systems [7], as well as in constructing level curves and normal forms of polynomials at critical points, as discussed in [4, 5, 6].

6. Main Achievements. The main result of the work performed is the creation of a rational parameterization algorithm adapted to specific types of algebraic curves. Unlike standard approaches, the proposed variant made it possible to simplify the computation process and make it more visual in the Maple environment. The obtained results can be used in further study of curve parameterization and automation of computations, which confirms the practical value of the work performed.

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