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TRANSPORT OF A DEGRADING SOLUTE IN A TWO-ZONE POROUS MEDIUM TAKING INTO ACCOUNT THE DECAY

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Abstract. This paper considers a mathematical model of the transport of a degrading solute in a two-zone porous medium, including active and passive zones. The model takes into account the processes of diffusion, adsorption and decay of the solute in both zones. To describe the dynamics of concentrations, a system of differential equations is used, which is solved numerically using the finite difference method. A numerical analysis of the influence of decay parameters on the behavior of concentration profiles is carried out.

Keywords: adsorption, decay of the solute, finite differences, mathematical model, multistage kinetics, porous medium.

Yemirilishni hisobga olgan holda ikki zonali g‘ovak muhitda degradatsiyalanuvchi modda ko‘chishi

Annotatsiya. Ushbu maqola aktiv va passiv zonalarni o'z ichiga olgan ikki zonali g'ovak muhitda yemiriluvchan modda ko’chishi matematik modeli qaralgan. Modelda har ikkala zonada erigan moddaning diffuziyasi, adsorbsiyalanishi va parchalanishi jarayonlarini hisobga olingan. Konsentratsiyalar dinamikasini tavsiflash uchun differensial tenglamalar sistemasi qo'llaniladi, va u chekli ayirmalar usuli yordamida yechilgan. Yemirilish parametrlarining kontsentratsiya profillariga ta'siri sonli tahlili qilingan.

Kalit so‘zlar: adsorbsiya, erigan moddaning yemirilishi, chekli ayirmalar, ko'p bosqichli kinetika, matematik model, g'ovak muhit.

Перенос деградирующего вещества в двухзонной пористой среде с учетом распада

Аннотация. В данной работе рассматривается математическая модель переноса деградирующего вещества в двухзонной пористой среде, включающей активную и пассивную зоны. Модель учитывает процессы диффузии, адсорбции и распада вещества в обеих зонах. Для описания динамики концентраций используется система дифференциальных уравнений, которая решается численно с помощью метода конечных разностей. Проведен численный анализ влияния параметров распада на поведение концентрационных профилей.

Ключевые слова: адсорбция, распад растворенного вещества, конечные разности, математическая модель, многоступенчатая кинетика, пористая среда.

Introduction. Modern environmental and engineering problems increasingly require a deep understanding of the processes of transport of pollutants and degrading solutes in porous media [1,2]. One of particular interest is the modeling of such processes in two-zone porous media, the structure of which includes both active zones with washable adsorption and a passive zone with non-washable adsorption [3, 4].

Some nonequilibrium models, that has been developed for modeling solute transport given in [5-8]. This is the "two-region' or "bi-continuum" transport model in which the sorption rate is thought to be limited by the rate at which solutes are transported by diffusion to the exchange sites. This conceptualization has led to physical nonequilibrium models that partition soil water into mobile (flowing) and stagnant (immobile or nonflowing) regions. The approach in effect assumes that the pore-water velocity distribution is bimodal: convective-dispersive transport is confined to only a fraction of the liquid-filled pores, while the remainder of the pores have stagnant water [5-9].

The transport of a degrading solutes in such systems is accompanied not only by diffusion and convection, but also by adsorption, as well as chemical or biochemical reactions of decay (degradation) [9-10]. Key degradation mechanisms are Biodegradation, Chemical Reactions, and Radioactive Decay [11-13]. In biodegradation, microorganisms naturally present in soil or groundwater break down organic pollutants. This process can be aerobic or anaerobic. Chemical Reactions, reactions like hydrolysis, oxidation, or reduction can transform or decompose contaminants. Radioactive Decay, for radioactive solutes, degradation is due to inherent radioactive disintegration. Taking these factors into account requires the construction and analysis of mathematical models capable of describing the dynamics of solute concentration, taking into account adsorption and degradation processes.

As noted in [4], the class of sorption (in the broad sense) processes includes filtration clarification of suspensions: filtration systems in which the extraction of suspension from the flow occurs inside the porous media. In such models, it is also noted that the deposition in the porous space of the globular structure of the filter has two forms - washable and non-washable. Accordingly, the filter zones are called active and passive [3,4]. The mathematical model of such processes consists of a balance equation and a kinetic equation for each zone [14, 15]. In [16, 17], the existence of two types of deposition was experimentally shown.

This article is devoted to the development and study of a mathematical model for the transport of a degrading solute in a two-zone porous medium. The features of the process flow, the influence of degradation characteristics on the distribution of the solute concentration are considered, and a numerical analysis is carried out based on typical examples.

Statement of the problem. A two-zone porous medium with active and passive zones is considered. Transport equation is as following [5]

$$\frac{\partial(\theta c)}{\partial e} = -div J_s - \frac{\partial c_a}{\partial t} - \frac{\partial c_p}{\partial t} - \theta\lambda_e c, \tag{1}$$

$$J_s = -\theta D\nabla c + \overset{\text{I}}{w}c, \tag{2}$$

where c is the concentration of the substance in the liquid, c_a is the concentration of the adsorbed substance in the active zone, c_p is the concentration of the adsorbed substance in the passive zone, J_s is the density of the substance flow in the active zone, θ is the porosity, λ_e is first order decay coefficient, $\overset{\text{I}}{w}$ is filtration velocity.

Substituting (2) into (1) gives

$$\frac{\partial(\theta c)}{\partial t} + div(\overset{\text{r}}{w}c) = div(\theta D\nabla c) - \frac{\partial c_a}{\partial t} - \frac{\partial c_p}{\partial t} - \theta\lambda_e c. \tag{3}$$

In the one-dimensional case from (3) we have

$$\frac{\partial(\theta c)}{\partial t} + \frac{\partial}{\partial x}(w_x c) = \frac{\partial}{\partial x}(\theta D\frac{\partial c}{\partial x}) - \frac{\partial c_a}{\partial t} - \frac{\partial c_p}{\partial t} - \theta\lambda_e c, \tag{4}$$

where w_x is the coordinate of the filtration velocity along the axis x.

If the medium is homogeneous, i.e. $\theta = const$, the filtration velocity is constant, and the coefficient of hydrodynamic dispersion is constant, from (4) we have

$$\frac{\partial c}{\partial t}+v_x\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial x^2}-\frac{1}{\theta}\frac{\partial c_a}{\partial t}-\frac{1}{\theta}\frac{\partial c_p}{\partial t}-\lambda_e c, \tag{5}$$

where $v_x=\frac{w_x}{\theta}$ is the component of the physical velocity of the liquid (solution).

For the concentration in the active zone, we have the following kinetic equations [4]

$$\frac{\partial c_a}{\partial t}=\beta_a\left(c-\frac{c_a}{c_{a0}}c_0\right)-\lambda_{ea}c_a, \tag{6}$$

where β_a is the coefficient characterizing the intensity of the adsorption of the substance in the active zone, c_{a0} is the maximum concentration upon reaching which, i.e. at $c_a=c_{a0}$ and when $c=c_0$ the adsorption of the substance in the active zone ceases, λ_{ea} is the coefficient of decay of the adsorbed substance in the active zone. Here together with (6) we use [18, 19]

$$\frac{\partial c_a}{\partial t}=\begin{cases}\beta_{ar}c-\lambda_{ea}c_a, & if\quad 0<c_a\le c_{ar},\\ \beta_{aa}c-\beta_{ad}c_a-\lambda_{ea}c_a, & if\quad c_{ar}<c_a<c_{a0},\\ 0, & if\quad c_a=c_{a0},\end{cases} \tag{7}$$

where β_{ar}, β_{aa}, β_{ad} are kinetic parameters, c_{ar} is the maximum concentration at which the “charging” effect is completed.

The kinetics of adsorption in the passive zone is taken as

$$\frac{\partial c_p}{\partial t}=\beta_n(c_p)c-\lambda_{ep}c_p, \tag{8}$$

where $\beta_p=\alpha(c_p)\beta_{p0}$,

$$\alpha(c_p)=\begin{cases}1, & if\quad c_p\le c_{p1},\\ \dfrac{c_{p1}}{c_p}, & if\quad c_{p1}<c_p<c_{p0},\\ 0, & if\quad c_p=c_{p0},\end{cases} \tag{9}$$

where c_{p1} is the maximum concentration at which the compaction effect begins, is the coefficient of decay of the adsorbed substance in the passive zone.

Taking into account (9), we write equation (8) in the form

$$\frac{\partial c_p}{\partial t}=\begin{cases}\beta_{a0}c-\lambda_{ep}c_p, & if\ 0<c_p\le c_{p1},\\ \dfrac{\beta_{p0}c_{p1}c}{c_p}-\lambda_{ep}c_p, & if\ c_{p1}<c_p<c_{p0},\\ 0, & if\ c_p=c_{p0}.\end{cases} \tag{10}$$

The problem consists of solving the system of equations (5), (7), (10) with the conditions

$$c(0,x)=0, c_a(0,x)=0, c_p(0,x)=0, \tag{11}$$

$$c(t,0)=c_0, c(t,\infty)=0, \tag{12}$$

where $c_0=const$.

Numerical solution. We use the finite difference method to solve problem (5), (7), (10) - (12) [20]. In the area $D=\{0\le x<\infty,\ 0\le t\le T\}$ we introduce a grid $\omega_{h\tau}$, where T is the value of time in which the process is under study. On the axis Ox the interval $[0,\infty)$ divided by step h and time interval $[0,T]$ divided by step τ. To approximate the equations, we introduce the following grid

$$\omega_{h\tau}=\left\{\left(x_i,t_j\right), x_i=ih, i=0,1,...,t_j=j\tau,\ j=\overline{0,J}, \tau=T/J\right\}$$

Instead of the functions $c(t,x)$, $c_a(t,x)$, $c_p(t,x)$ we consider specific functions whose values at the nodes (x_i,t_j) defined as c_i^j, $c_{i,a}^j$, $c_{i,p}^j$, respectively.

Equation (5) we approximate in the following form on the grid $\omega_{h\tau}$

$$\frac{c_i^{j+1}-c_i^j}{\tau}+v_x\frac{c_i^{j+1}-c_{i-1}^{j+1}}{h}=D\frac{c_{i+1}^{j+1}-2c_i^{j+1}+c_{i-1}^{j+1}}{h^2}-\frac{1}{\theta}\frac{c_{a,i}^{j+1}-c_{a,i}^j}{\tau}$$
$$-\frac{1}{\theta}\frac{c_{p,i}^{j+1}-c_{p,i}^j}{\tau}-\lambda_e c_i^{j+1}. \tag{13}$$

The differential schemes for equations (7) and (10) are as follows:

$$\frac{c_{a,i}^{j+1}-c_{a,i}^j}{\tau}=\begin{cases}\beta_{ar}vc_i^j-\lambda_{ea}c_{a,i}^j, & 0<c_{a,i}^j\le c_{ar},\\ \beta_{aa}vc_i^j-\beta_{ad}c_{a,i}^j-\lambda_{ea}c_{a,i}^j, & c_{ar}<c_{a,i}^j<c_{a0},\\ 0, & c_{a,i}^j=c_{a0}.\end{cases} \tag{14}$$

$$\frac{c_{p,i}^{j+1}-c_{p,i}^j}{\tau}=\begin{cases}\beta_{p0}c_i^j-\lambda_{ep}c_{p,i}^j, & 0<c_{p,i}^j\le c_{p1},\\ \dfrac{\beta_{p0}c_{p1}c_i^j}{c_{p,i}^j}-\lambda_{ep}c_{p,i}^j & c_{p1}<c_{p,i}^j<c_{p0},\\ 0, & c_{p,i}^j=c_{p0}.\end{cases} \tag{15}$$

Initial and boundary conditions (11), (12) have the form

$$c_{i,a}^0=0,\ c_{i,p}^0=0,\ c_i^0=0,\ i=\overline{0,I},,\ c_i^j=c_0,\ i=0,\ j=\overline{0,J}, \tag{16}$$

where I is sufficiently large value, for which $c_I^j=0$ is true.

Numerical algorithm is as follows. First by using the initial conditions we find c_i^j, $c_{i,a}^j$, $c_{i,p}^j$, in zero time layer. Then, use (15) to find $c_{i,p}^{j+1}$ and (14) for $c_{i,a}^{j+1}$. Finally, substituting $c_{i,a}^j$ and $c_{i,p}^j$ into (13), by using Thomas' algorithm we determine c_i^j on the $(j+1)$ th layer of the grid $\omega_{h\tau}$.

Results and discussion. Figure 1 shows three concentration fields, $c/c_0(a)$, $c_a(b)$, $c_p(c)$. It is evident from the figure that over time, all three concentration fields move inside the porous medium. Here, the solute transport occurs faster than adsorption. The change in the concentration of the adsorbed substance in the active and passive zones occurs with a slight delay compared to the solute transport.

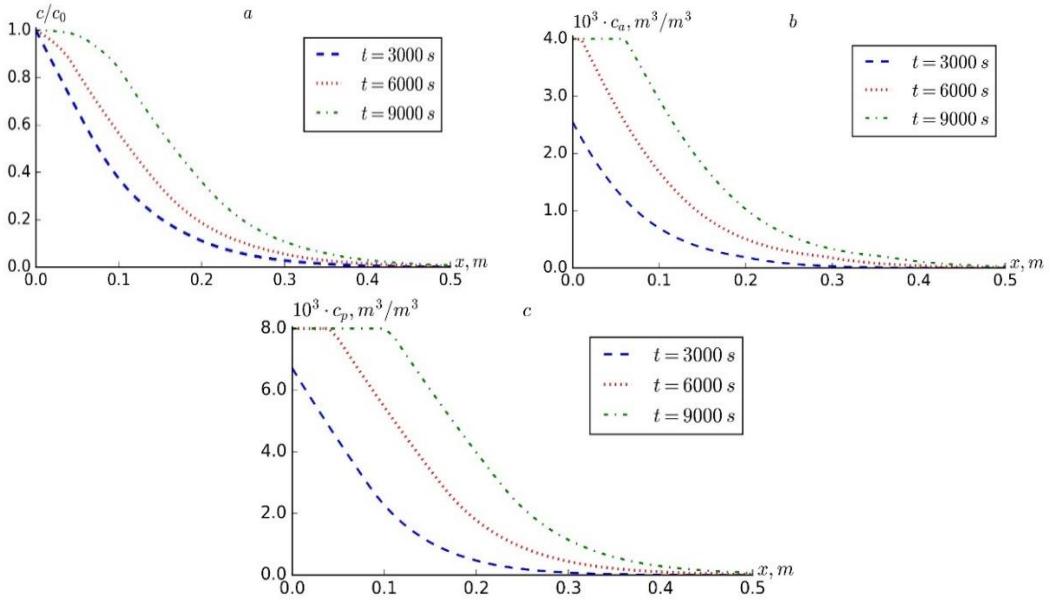


FIGURE 1. Profiles of $c/c_0(a)$, $c_a(b)$, $c_p(c)$, at $\lambda_e=\lambda_{ea}=\lambda_{ep}=0$.

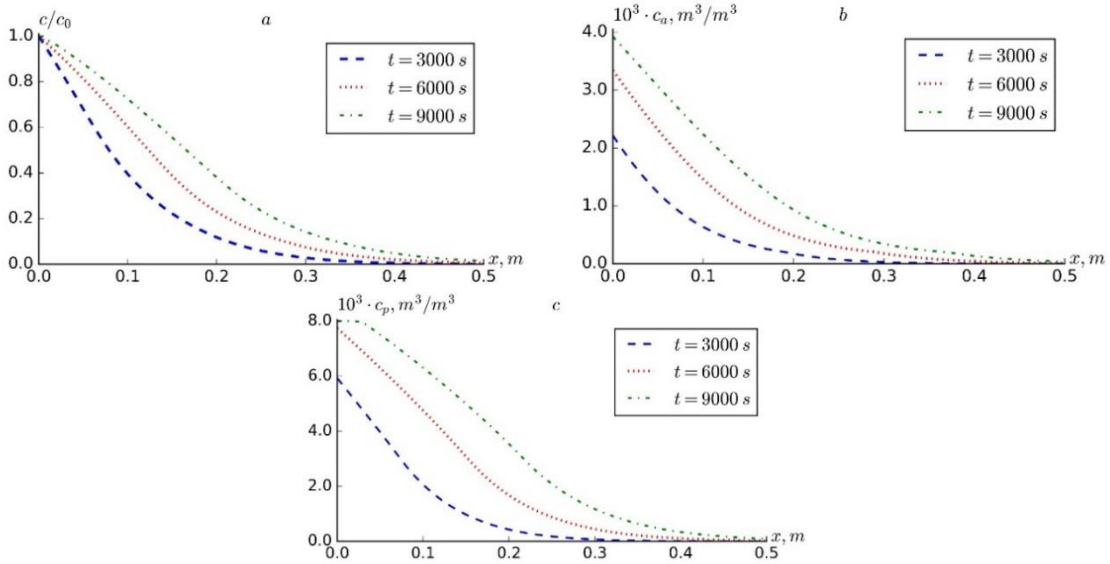


FIGURE 2. Profiles of $c / c_0 (a)$, $c_a (b)$, $c_p (c)$, at $\lambda_e = \lambda_{ea} = \lambda_{ep} = 1 \cdot 10^{-4}$ s^{-1}.

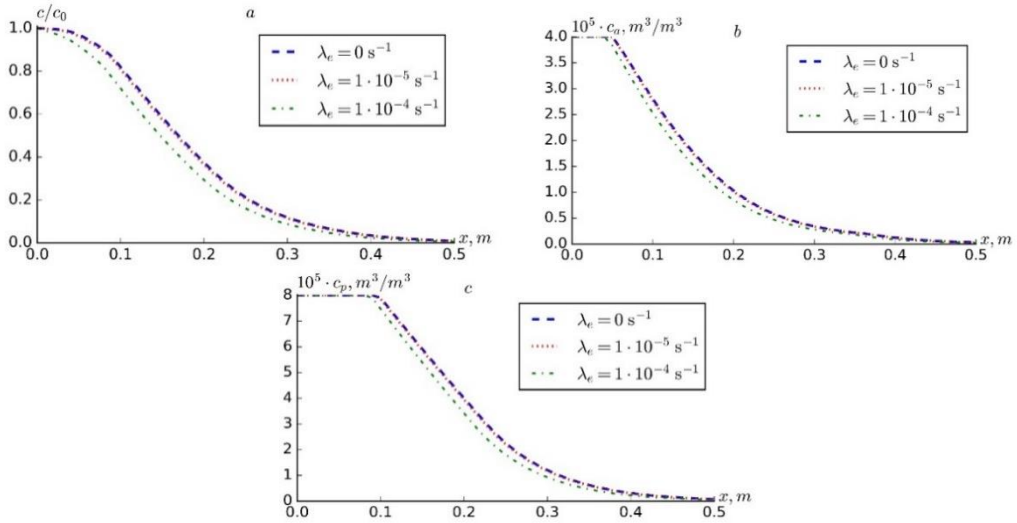


FIGURE 3. Profiles of $c / c_0 (a)$, $c_a (b)$, $c_p (c)$, at $t = 9000$ s and different values of λ_e.

At $t = 3000$ s concentration c / c_0 reaches approximately point $x \approx 0.35$ m, while the concentration of c_a and c_p reaches $x \approx 0.3$ m. At $t = 6000$ s concentration c_p reaches its maximum value not only at the point $x = 0$, but also at points close to this point, however c_a reaches its maximum only at point $x = 0$. This can be explained by the fact that the adsorption intensity in the passive zone is greater than in the active zone, and in the passive zone there is no desorption, which is possible in the active zone. At $t = 9000$ s, it is clear that the concentration of c_p reaches its maximum value in the passive zone up to a distance of about $x \approx 0.12$ m, while the concentration c_a reaches equilibrium to a distance $x \approx 0.08$ m. Figure 2 shows the results for the case where the decay coefficients are not zero. In Figure 2, compared to Figure 1, where $\lambda_e = \lambda_{ea} = \lambda_{ep} = 0$, it is clear that all three areas have decreased. Over time, one can see that this difference becomes more noticeable. Comparing Figures 1b and 2b, one can see that at $t = 9000$ s in Figure 2-b c_a did not reach its maximum value at point $x = 0$, whereas in Figure 1b this value was maximum at $t = 6000$ s. In Figure 2-c, it is clear that the value c_p reaches its maximum value only at t=9000 s, and compared to Figure 1c, there are also significant delays. All this is explained by a significant decrease in the concentration values as a result of decay.

Figure 3 shows the results for three values of the decay coefficient λ_e involved in the balance equation. It is evident that the case with $\lambda = 1 \cdot 10^{-5}$ is not significantly different from the case with $\lambda = 0$. At $\lambda = 1 \cdot 10^{-4}$ it is evident that the value c / c_0 decreases significantly (Fig. 3a). This, in turn, led to a decrease c_a and c_p (Fig. 3b, c). Figure 4 shows the results for different values of λ_{ea}. It is clear that increasing λ_{ea} leads to a significant decrease in c_a (Fig. 4b). The values of c / c_0 and c_p, on the contrary, increased. Figure 5 shows the results for different values of λ_{ep}, where increasing its value results in a decrease c_p up to $x \approx 0,25$, and then a very slight increase, while c_a increases at various points, while c / c_0 decreases near $x = 0$, and then increases.

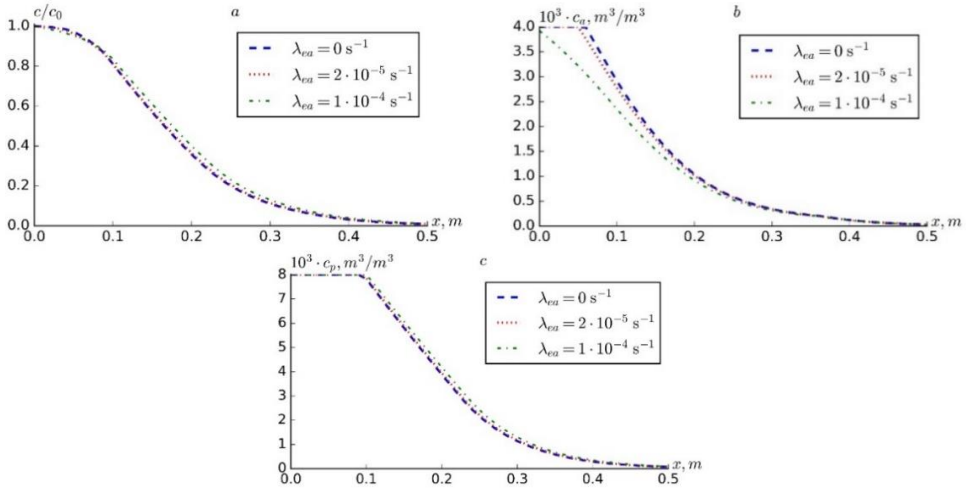


FIGURE 4. Profiles of c / c_0 (*a*), c_a (*b*), c_p (*c*), at $t = 9000$ s and different values of λ_{ea}.

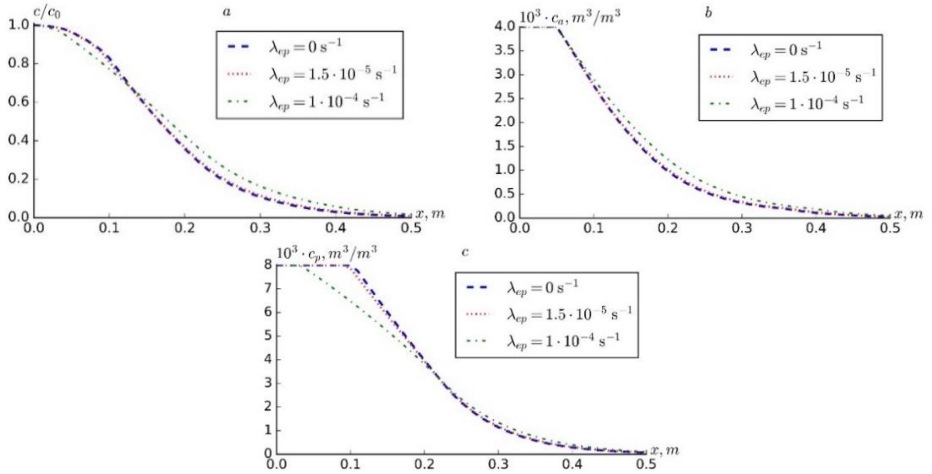


FIGURE 5. Profiles of c / c_0 (*a*), c_a (*b*), c_p (*c*), at $t = 9000$ s and different values of λ_{ep}.

Conclusion. The article developed and investigated a mathematical model of the transfer of a degrading substance in a two-zone porous medium. Taking into account the active and passive zones with different kinetic characteristics of the sedimentation and decomposition of the substance allowed us to obtain a more realistic picture of the process. The results of numerical modeling showed that adsorption processes occur with a delay compared to the transfer of the dissolved substance, and degradation significantly affects the shape and speed of propagation of the concentration front.

References

1. Baqer Y., Chen X. A review on reactive transport model and porosity evolution in the porous media. Environ Sci Pollut Res 29, 47873–47901 (2022). https://doi.org/10.1007/s11356-022-20466-w.

2. Liu Z.; Gan Y.; Luo J.; Luo,X.; Ding C.; Cui Y. Current Status of Emerging Contaminant Models and Their Applications Concerning the Aquatic Environment: A Review. Water 2025, 17, 85.
3. Веницианов Е.В., Рубинштейн Р.Н. Динамика сорбции из жидких сред. – М.: Наука, 1983 – 237 с.
4. Веницианов Е.В., Рубинштейн Р.Н., Сенявин М.М. О возможности распространения теории динамики сорбции на расчет осветления воды зернистыми фильтрами // Докл. АН СССР. 1970. 195, № 3. с. 658-661.
5. Coats K.H., B.D. Smith. 1964. Dead-end pore volume and dispersion in porous media. Soc. Petrol. Eng. J. 4:73-84.
6. Gaudet J.P., H. Jegat, G. Vachaud, and P.J. Wierenga. 1977. Solute transfer, with exchange between mobile and immobile water, through unsaturated sand. Soil Sci. Soc. Am. J. 41:665-671.
7. Van Genuchten, M.Th., and Cleary R.W. 1979. Movement of solutes in soil: Computer-simulated and laboratory results, p. 349386. In G.H. Bolt (ed.) Soil chemistry, B. Physicochemical models. Elsevier, Amsterdam.
8. Nkedi-Kizza P., J.W. Biggar, H.M. Selim, M.Th. van Genuchten, P.J. Wierenga, J.M. Davidson, and D.R. Nielsen. 1984. On the equivalence of two conceptual models for describing ion exchange during transport through an aggregated oxisol. Water Resour. Res. 20:1123-1130.
9. Van Genuchten M. Th. and Wagenet R. J., Two – Site/Two – Region Models for Pesticide Transport and Degradation: Theoretical Development and Analytical Solutions. Soil Sci. Soc. Am. J. 53, 1989. 1303 – 1310.
10. Ogram A.V., Jessup R. E., Ou L. T., and Rao P. S. "Effects of sorption on biological degradation rates of (2,4-dichlorophenoxy) acetic acid in soils," Applied and Environmental Microbiology, vol. 49, no. 3, pp. 582–587, Mar. 1985, https://doi.org/10.1128/aem.49.3.582-587.1985.
11. Mover J. R., Hance R. J., McKone C. E. "The effect of adsorbents on the rate of degradation of herbicides incubated with soil," Soil Biology and Biochemistry, vol. 4, no. 3, pp. 307–311, Aug. 1972.
12. Brusseau M. L., Xie L. H., Li L. "Biodegradation during contaminant transport in porous media: 1. mathematical analysis of controlling factors," Journal of Contaminant Hydrology, vol. 37, no. 3, pp. 269–293, Apr. 1999, https://doi.org/10.1016/S0169-7722(99)00005-4.
13. Kammouri S. A., M. El Hatri, and J. M. Crolet, "Modeling Contaminant Transport and Biodegradation in a Saturated Porous Media," in Computational Methods for Flow and Transport in Porous Media, J. M. Crolet, Ed. Dordrecht Netherlands: Springer Netherlands, 2000, pp. 237–243.
14. B. Kh. Khuzhayorov, J. M. Makhmudov, B. M. Fayziev, and T. I. Begmatov, "Some Model of a Suspension Filtration in a Porous Media That Accounts for the Two-Zone and Multistage Character of Deposition Kinetics," Journal of Applied and Industrial Mathematics, vol. 14, no. 3, pp. 513–523, Aug. 2020, https://doi.org/10.1134/S1990478920030102
15. Khuzhayorov B., Makhmudov J., Fayziev B., Begmatov T., Sagdullaev O. Model of two-component suspension fltration in a porous medium with multistage deposition kinetics, AIP Conf. Proc., 2637, 04001, (2022); doi 10.1063/5.0118455.
16. Ma E.; Ouahbi T.; Wang H.; Ahfir N.D.; Alem A.; Hammadi A. Modeling of retention and re-entrainment of mono- and poly-disperse particles: Effects of hydrodynamics, particle size and interplay of different-sized particles retention. Sci. Total Environ. 2017, 596–597, 222–229.
17. Ma E.; Ouahbi T.; Wang H.; Ahfir N.D.; Alem A.; Hammadi A. Modeling of the transport and deposition of polydispersed particles: Effects of hydrodynamics and spatiotemporal evolution of the deposition rate. Environ. Pollut. 2017, 237, 1011–1022.
18. Gitis V., Rubinstein I., Livshits M., Ziskind M. Deep – bed filtration model with multistage deposition kinetics. Chemical Engeneering Journal. – 2010. №. 163. Pp. 78 – 85.
19. Fayziev B., Ibragimov G., Khuzhayorov B., Alias I. A., Numerical study of suspension filtration model in porous medium with modified deposition kinetics. Symmetry, 12 № 5 (2020), 696.
20. Самарский А. А. Теория разностных схем. – М.: Наука, 1977. – 656 с.