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NONLINEAR RANDOM PARAMETRIC OSCILLATIONS OF A VIBRATION-PROOFED MECHANICAL SYSTEM

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Abstract. In this paper, the problem of nonlinear random parametric oscillations of a vibration-protected mechanical system is considered. Nonlinear ambiguous functions characterizing the dissipative properties of materials of elastic damping elements of a mechanical system and a dynamic absorber are taken into account using the method of statistical linearization in linear form. Using the Ito method, an analytical expression for the mean square values of the vibration-protected system is obtained.

Key words: mechanical system, dynamic damper, hysteresis, random, parametric vibration, stochastic process, mean square value.

Tebranishlardan himoyalanuvchi mexanik sistemaning chiziqlimas tasodifiy parametrik tebranishlari

Annotatsiya. Ushbu ishda tebranishlardan himoyalanuvchi mexanik sistemani chiziqlimas tasodifiy parametrik tebranishlari dinamikasini tekshirish haqidagi masala qaralgan. Mexanik sistema va dinamik so‘ndirgichni elastik dempferlovchi elementlari materiallarining dissipativlik xossalarini ifodalovchi chiziqlimas bir qiymatli bo‘lmagan funksiyalar statistik chiziqlilashtirish usuli yordamida chiziqli funksiyalar ko‘rinishda hisobga olingan. Ito usuli yordamida tebranishlardan himoyalanuvchi sistema o‘rta kvadratik qiymatlarini analitik ifodasi aniqlangan.

Kalit so‘zlar: mexanik sistema, dinamik so‘ndirgich, gisterezis, tasodifiy, parametrik tebranish, stoxastik jarayon, o’rta kvadratik qiymat.

Нелинейные случайные параметрические колебания виброзащищаемой механической системы

Аннотация. В данной работе рассмотрена задача о нелинейных случайных параметрических колебаниях виброзащищаемой механической системы. Нелинейные неоднозначные функции, характеризующие диссипативные свойства материалов упругодемпфирующих элементов механической системы и динамического гасителя, учитываются с помощью метода статистической линеаризации в линейной форме. Используя метода Ито, получено аналитическое выражение среднее квадратических значений виброзащищаемой системы.

Ключевые слова: механическая система, динамический гаситель, гистерезис, случайный, параметрическое колебание, стохастический процесс, среднее квадратическое значение.

Introduction. Damping harmful vibrations of mechanical systems under the influence of random parametric excitations is a pressing problem. In this regard, mathematical modeling of the elements of mechanical systems, taking into account the elastic dissipative characteristics of the hysteresis type, and determining their optimal parameters are important issues.

In works [1-7] the use of stochastic methods in solving the problems of protection of various types of mechanical systems from harmful vibrations is considered.

In the work [8], the issue of dynamics and stability of nonlinear vibrations of a hysteresis-type elastic dissipative characteristic mechanical system combined with a liquid section dynamic absorber under the influence of random parametric excitations was considered. Using the method of stochastic averaging, the system of differential equations of motion is presented as a system of Ito equations.

In the work [9], the behavior of a mechanical system with lumped mass in dynamic and kinematic processes was studied. The differential equations of motion were created, the mean square values of the system and base acceleration were found through the spectral density function in white noise excitations, and a numerical analysis was performed to select the optimal parameters.

The work [10] is devoted to the study of vibrations of a mechanical system under random excitations. The non-stationary effect is obtained through the Green's function. Analytical expressions were obtained to determine the mathematical expectation and correlation function.

In the work [11], a new approach was considered to describe energy loss in non-stationary vibrations of mechanical systems through the hysteresis loop. A mathematical model of the hysteresis loop was created, its parameters were determined using experimental data for stationary vibrations of the system.

The work [12] considered the issues of stability of longitudinal and transverse vibrations of the rod and beam under the influence of external forces. In this case, a random function was selected and the values at which stability conditions were determined through the spectral density of longitudinal and transverse vibrations in white noise excitations.

In the work [13], the method of Lagrange multipliers was used to select the optimal parameters of a mechanical system with four degrees of freedom under the influence of harmonic excitations. The results for damping harmful vibrations were obtained and the amplitude-frequency characteristic was compiled by numerical calculation. Recommendations for choosing optimal parameters for effective damping of vibrations are given.

In the work [14], the mean square value and stability in random parametric excitations of mechanical systems were studied. Ito's differential equation is derived by the stochastic averaging method, and it is shown that the obtained results are also appropriate for the stability of system motion. System motion stability conditions are expressed depending on the spectral density of vibrations. Changes of stability and unstability fields depending on the damping coefficient were numerically analyzed.

In the work [15], the question of stability of mechanical systems with hysteresis-type connections in random excitations was studied. In this case, the differential equations of motion of the systems were reduced to the Ito differential equation using the stochastic averaging method, and the stability of motion was checked using the Lyapunov method. In order to check the reliability of stability conditions, the issues were solved and analyzed.

In works [16-18] mathematical modeling of nonlinear mechanical systems, study of dynamics and checking of stability of motions and instructions for selection of parameters corresponding to stable motions, in particular mechanical system materials are given. Modal mass and modal stiffnness expressions are analytically expressed. With their help, the issues of choosing and modeling system materials were also solved.

Problem statementand solution methodology. This work deals with the study of the dynamics of random parametric oscillations of a mechanical system with hysteresis-type elastic dissipative characteristics under the influence of excitations, together with a dynamic damper. The dissipative properties of the materials of the elastic damping elements of the mechanical system and the dynamic damper are obtained in the hysteresis type. The relationships between stress and deformation are obtained in terms of nonlinear single-valued functions and expressed in linear functions using the statistical linearization method.The system of differential equations of motion of a mechanical system protected from vibrations is as follows:

$$m_1\ddot{x}_1 + c_1(1-\eta_1+i\eta_2)x_1 - c_2(1-\nu_1+i\nu_2)x_2 = -c_1\xi_0(t)x_1(t);$$
$$m_2\ddot{x}_1 + m_2\ddot{x}_2 + c_2(1-\nu_1+i\nu_2)x_2 = 0, \quad (1)$$

where m_1, m_2 are the masses of the protected object and the dynamic absorber, respectively, x_1, x_2 are the displacement coordinates of the protected object and the dynamic absorber; c_1, c_2 are elasticity coefficients of the object to be protected and the dynamic absorber; $i^2 = -1$; η_1, η_2 are linearization coefficients representing the dissipative properties of the material of the elastic damping element of the object; ν_1, ν_2 are linearization coefficients representing the dissipative properties of the material of the elastic damping element of the dynamic absorber; $\xi_0(t)$ is a variable representing a stationary normal random process.

Let's express the differential equations of motion of the mechanical system as follows:

$$\ddot{x}_1 + \omega_1^2(1-\eta_1+i\eta_2)x_1 - \mu\omega_2^2(1-\nu_1++i\nu_2)x_2 = -\omega_1^2\xi_0(t)x_1(t);$$
$$\ddot{x}_1 + \ddot{x}_2 + \omega_2^2(1-\nu_1+i\nu_2)x_2 = 0, \quad (2)$$

where

$$\mu = \frac{m_2}{m_1}; \omega_1^2 = \frac{c_1}{m_1}; \omega_2^2 = \frac{c_2}{m_2}.$$

The solution of the system of differential equations (2) is looked for in the following form:

$$x_1 = Ae^{i\omega t} + Be^{-i\omega t};$$
$$x_2 = Ce^{i\omega t} + De^{-i\omega t}, \quad (3)$$

where $\omega -$ is the frequency of vibrations; $A = A(t), B = B(t), C = C(t), D = D(t)$ are slowly variable functions, and the amplitude value of random parametric excitations of the mechanical system is $\langle x_{1a}\rangle = 2\sqrt{\langle A(t)\rangle\langle B(t)\rangle}$ satisfies the condition [5,6].

Taking into account that A, B, C, D are slowly variable functions, if the variables (3) and their respective derivatives are put into the system of differential equations (2), it is possible to get the following system of differential equations of the first order:

$$\dot{A} = \frac{1}{2i\omega}\{[\omega^2 - \omega_1^2\xi_0(t) - \omega_1^2(1-\eta_1+i\eta_2)]\left(A + Be^{-2i\omega t}\right) + \\ +\mu\omega_2^2(1-\nu_1+i\nu_2)\left(C + De^{-2i\omega t}\right)\};$$
$$\dot{B} = \frac{-1}{2i\omega}\{[\omega^2 - \omega_1^2\xi_0(t) - \omega_1^2(1-\eta_1+i\eta_2)]\left(Ae^{2i\omega t} + B\right) + \\ +\mu\omega_2^2(1-\nu_1+i\nu_2)\left(Ce^{2i\omega t} + D\right)\}; \quad (4)$$
$$\dot{C} = \frac{\hat{}}{2i\omega}\{[\omega^2 + (\mu+1)\omega_2^2(1-\nu_1+i\nu_2)]\left(De^{-2i\omega t} + C\right) + \\ +[-\omega_1^2\xi_0(t) + \omega_1^2(1-\eta_1+i\eta_2)]\left(A + Be^{-2i\omega t}\right)\};$$
$$\dot{D} = \frac{\hat{}}{2i\omega}\{[\omega^2 + (\mu+1)\omega_2^2(1-\nu_1+i\nu_2)]\left(Ce^{2i\omega t} + D\right) + \\ +[-\omega_1^2\xi_0(t) + \omega_1^2(1-\eta_1+i\eta_2)]\left(Ae^{2i\omega t} + B\right)\}.$$

In stochastic processes, the variables of the system of equations (4) satisfy the Ito equations [6]. Therefore, use the stochastic averaging method and change the system of equations (4) to the system of Ito equations.

$$\frac{d\langle A\rangle}{dt} = p_1\langle A\rangle + \frac{\pi}{2}p_5^2\left(S(0) - i\psi(2\omega)\right)\langle A\rangle + p_2\langle C\rangle;$$
$$\frac{d\langle B\rangle}{dt} = -p_1\langle B\rangle - \frac{\pi}{2}p_5^2\left(S(2\omega) - S(0)\right)\langle B\rangle - p_2\langle D\rangle; \quad (5)$$

$$\frac{d\langle C\rangle}{dt} = p_4\langle A\rangle + \frac{\pi}{2}p_5^2\big(S(0) - i\psi(2\omega)\big)\langle A\rangle + p_3\langle C\rangle;$$

$$\frac{d\langle D\rangle}{dt} = -p_4\langle B\rangle - \frac{\pi}{2}p_5^2\big(S(2\omega) - S(0)\big)\langle B\rangle - p_3\langle D\rangle,$$

Where

$$p_1 = \frac{\omega^2 - \omega_1^2(1-\eta_1+i\eta_2)}{2i\omega}; p_2 = \frac{\mu\omega_2^2(1-\nu_1+i\nu_2)}{2i\omega};\ p_3 = \frac{\omega^2+(1+\mu)\omega_2^2(1-\nu_1+i\nu_2)}{2i\omega};$$

$$p_4 = \frac{\omega_1^2(1-\eta_1+i\eta_2)}{2i\omega}; p_5 = \frac{-\omega_1^2}{2i\omega}.$$

and $S(0), S(2\omega), \psi(2\omega)$ are defined as the spectral density of a stationary normal random process $\xi_0(t)$ as follows [5,6]:

$$S(2\omega) = \frac{1}{\pi}\int_{-\infty}^{0} R(\tau)cos\omega\tau d\tau\,; \psi(2\omega) = \frac{1}{\pi}\int_{-\infty}^{0} R(\tau)sin\omega\tau d\tau,$$

$R(\tau) = E[\xi_{0n}(t)\xi_{0m}(t+\tau)] = \langle\xi_{0n}(t)\xi_{0m}(t+\tau)\rangle -$ is correlation function.

Let's look for the solution of the system of equations (5) as follows:

$$A = A_0e^{\lambda t}; B = B_0e^{\lambda t}; C = C_0e^{\lambda t}; D = D_0e^{\lambda t}; (6)$$

where A_0, B_0, C_0, D_0 are the amplitude values of the random parametric excitations of the mechanical system; λ is a characteristic number.

Let's put the solutions (6) into the system of equations (5) and from the condition of having a non-zero solution, the determinant form of the characteristic equation of the system can be formed as follows:

$$\begin{bmatrix} a_{11}+\lambda & \cdots & a_{14} \\ \vdots & \ddots & \vdots \\ a_{41} & \cdots & a_{44}+\lambda \end{bmatrix} = 0, (7)$$

where

$$a_{11} = p_1 + \frac{\pi}{2}p_5^2[S(0) + i\psi(2\omega)]; a_{12} = 0; a_{13} = p_2; a_{14} = 0;$$

$$a_{21} = 0; a_{22} = -p_1 - \frac{\pi}{2}p_5^2[S(2\omega) - S(0)]; a_{23} = 0; a_{24} = -p_2;$$

$$a_{31} = p_4 + \frac{\pi}{2}p_5^2[S(0) + i\psi(2\omega)]; a_{32} = 0; a_{33} = p_3; a_{34} = 0;$$

$$a_{41} = 0; a_{42} = -p_4 - \frac{\pi}{2}p_5^2[S(2\omega) - S(0)]; a_{43} = 0; a_{44} = -p_3.$$

Using the determinant (7), it is possible to form the following two characteristic equations:

$$\lambda^4 - (a_{11}+a_{22})\lambda^3 + (a_{11}a_{22} + (a_{42}-a_{31})p_2 - p_3^2)\lambda^2 +$$
$$+\Big((a_{11}+a_{22})p_3^2 + (a_{22}a_{31} - a_{11}a_{42})p_2 - (a_{31}+a_{42})p_2p_3\Big)\lambda -$$
$$-a_{11}a_{22}p_3^2 + (a_{11}a_{42} + a_{22}a_{31})p_2p_3 - a_{31}a_{42}p_2^2 = 0. (8)$$

Result and discussion. The characteristic equations are quadratic equations. Let's identify their roots.

$$\lambda_1 = \tfrac{1}{2}(a_{11}+p_3) + \tfrac{1}{2}\sqrt{(a_{11}-p_3)^2 + 4a_{31}p_2};$$

$$\lambda_2 = \tfrac{1}{2}(a_{11}+p_3) - \tfrac{1}{2}\sqrt{(a_{11}-p_3)^2 + 4a_{31}p_2}; (9)$$

$$\lambda_3 = \tfrac{1}{2}(a_{22}-p_3) + \tfrac{1}{2}\sqrt{(a_{22}+p_3)^2 - 4a_{42}p_2};$$

$$\lambda_4 = \tfrac{1}{2}(a_{22}-p_3) - \tfrac{1}{2}\sqrt{(a_{22}+p_3)^2 - 4a_{42}p_2}.$$

Using Moivre's formula for expressions involving roots of the characteristic equation (9), we determine their values. In this case, the roots of (9) will be as follows:

$$\lambda_1 = \frac{1}{2}\Big(p_1 + \frac{\pi}{2}p_5^2\big(S(0) + i\psi(2\omega)\big) + p_3\Big) + \frac{1}{8\omega^2}(\alpha_1 + i\beta_1);$$

$$\lambda_2 = \frac{1}{2}\Big(p_1 + \frac{\pi}{2}p_5^2\big(S(0) + i\psi(2\omega)\big) + p_3\Big) - \frac{1}{8\omega^2}(\alpha_1 + i\beta_1); \quad (10)$$

$$\lambda_3 = \frac{1}{2}\Big(-p_1 - \frac{\pi}{2}p_5^2\big(S(2\omega) - S(0)\big) - p_3\Big) + \frac{1}{8\omega^2}(\alpha_2 + i\beta_2);$$

$$\lambda_4 = \frac{1}{2}\Big(-p_1 - \frac{\pi}{2}p_5^2\big(S(2\omega) - S(0)\big) - p_3\Big) - \frac{1}{8\omega^2}(\alpha_2 + i\beta_2),$$

where

$$a_1 = 16\{[\eta_2^2 - (1-\eta_1)^2]\omega_1^4 - 6[(1-\eta_1)(1-\nu_1) + \eta_2\nu_2]\Big(\mu + \frac{1}{3}\Big)\omega_2^2\omega_1^2 -$$
$$-(1+\mu)^2[(1-\nu_1)^2 - \nu_2^2]\omega_2^4\}\omega^2 + 8\omega_1^4\pi \times$$
$$\times \Big([(1-\eta_1)\omega_1^2 - (1-\nu_1)\omega_2^2(\mu-1)]\psi(2\omega) + [\eta_2\omega_1^2 - \nu_2(\mu -$$
$$1)\omega_2^2]S(0)\Big) \times$$
$$\times \omega - \Big[\big(\psi(2\omega)\big)^2 - S(0)^2\Big]\pi^2\omega_1^8;$$

$$b_1 = -32\{(1-\eta_1)\eta_2\omega_1^4 + 3[\eta_2(1-\nu_1) + \nu_2(1-\eta_1)]\Big(\mu + \frac{1}{3}\Big)\omega_2^2\omega_1^2 +$$
$$+(1+\mu)^2\omega_2^4(1-\nu_1)\nu_2\}\omega^2 + 8\omega_1^4\pi\omega \times$$
$$\times \Big([\eta_2\omega_1^2 - \nu_2\omega_2^2(\mu-1)]\psi(2\omega) + [-(1-\eta_1)\omega_1^2 + (1-\nu_1)(\mu -$$
$$1)\omega_2^2]S(0)\Big)$$
$$+2\omega_1^8\pi^2 S(0)\psi(2\omega);$$

$$a_2 = 16\{[\eta_2^2 - (1-\eta_1)^2]\omega_1^4 - 6[(1-\eta_1)(1-\nu_1) - \eta_2\nu_2]\Big(\mu + \frac{1}{3}\Big)\omega_1^2\omega_2^2 -$$
$$-(1+\mu)^2[(1-\nu_1)^2 - \nu_2^2]\omega_2^4\}\omega^2 + 8\omega_1^4\pi \times$$
$$\times \Big(\eta_2\omega_1^2 - \nu_2\omega_2^2(\mu-1)\Big)\big(S(2\omega) - S(0)\big)\omega + \pi^2\omega_1^8\big(S(0) - S(2\omega)\big)^2;$$

$$b_2 = -32\{(1-\eta_1)\eta_2\omega_1^4 + 3[\eta_2(1-\nu_1) + \nu_2(1-\eta_1)]\Big(\mu + \frac{1}{3}\Big)\omega_2^2\omega_1^2 +$$
$$+(1+\mu)^2\omega_2^4(1-\nu_1)\nu_2\}\omega^2 + 8\omega_1^4\pi\Big(-(1-\eta_1)\omega_1^2 + (1-\nu_1)\omega_2^2(\mu-1)\Big) \times$$
$$\times \big(S(2\omega) - S(0)\big)\omega.$$

The roots of the identified characteristic equation allow for the examination of the mean square expectations and the stability of a mechanical system protected against damped oscillations.

According to stability theory, for a damped motion to be asymptotically stable, it is sufficient that the real parts of the characteristic equation's roots are negative. Based on this, we determine the boundary between stable and unstable motions

$$-\omega_1^2\nu_2 + \pi p_5^2\omega S(0) + (1+\mu)\omega_2^2\eta_2 + \frac{1}{2\omega}\alpha_1 = 0;$$

$$-\omega_1^2\nu_2 + \pi p_5^2\omega S(0) + (1+\mu)\omega_2^2\eta_2 - \frac{1}{2\omega}\alpha_1 = 0; \quad (11)$$

$$\omega_1^2\nu_2 - \pi p_5^2\omega\big(S(2\omega) - S(0)\big) - (1+\mu)\omega_2^2\eta_2 + \frac{1}{2\omega}\alpha_2 = 0;$$

$$\omega_1^2\nu_2 - \pi p_5^2\omega\big(S(2\omega) - S(0)\big) - (1+\mu)\omega_2^2\eta_2 - \frac{1}{2\omega}\alpha_2 = 0.$$

By squaring the equations (11), we can reduce them to the following two equations.

$$8\omega^2\left(-\omega_1^2\nu_2+\pi p_5^2\omega S(0)+(1+\mu)\omega_2^2\eta_2\right)^2-(a_1^2+b_1^2)^{\frac{1}{2}}\times$$
$$\times\left((a_1^2+b_1^2)^{\frac{1}{2}}+a_1\right)=0; \quad (12)$$
$$8\omega^2\left(\omega_1^2\nu_2-\pi p_5^2\omega\left(S(2\omega)-S(0)\right)-(1+\mu)\omega_2^2\eta_2\right)^2-(a_2^2+b_2^2)^{\frac{1}{2}}\times$$
$$\times\left((a_2^2+b_2^2)^{\frac{1}{2}}+a_2\right)=0.$$

In order to determine the mean square values from expressions (12), we use the following in them Spectral density is chosen as follows:

$$S(\omega)=\frac{\sigma_\xi^2\chi q^3}{\pi[(q^2-\omega^2)^2+(2\chi q\omega)^2]},$$

where $\sigma_\xi-$ is the mean square value of the base excitation; $\chi-$ is parameter describing the width of vibration spectrum; $q-$ is dominant frequency.

From the expressions (12), it is possible to determine the mean square values σ_{x_1},$\wedge\,\sigma_{x_2}$ for the considered system.

N.Davidenko's hypothesis is used for linearization of hysteresis-type nonlinearity. According to it, the linearization coefficients are as follows [7]:

$$\eta_1=\frac{\eta(n-1)\Gamma\left(\frac{n+3}{2}\right)}{\pi(n+1)B(n,n)}2^{\frac{3-n}{2}}{\sigma_{x_1}}^{n-1};\eta_2=\frac{\eta(n-1)\Gamma\left(\frac{n+3}{2}\right)}{\pi(n+1)n}2^{\frac{3n+1}{2}}{\sigma_{x_1}}^{n-1};$$
$$\nu_1=\frac{\eta_0(n-1)\Gamma\left(\frac{n+3}{2}\right)}{\pi(n+1)B(n,n)}2^{\frac{3-n}{2}}{\sigma_{x_2}}^{n-1};\nu_2=\frac{\eta_0(n-1)\Gamma\left(\frac{n+3}{2}\right)}{\pi(n+1)n}2^{\frac{3n+1}{2}}{\sigma_{x_2}}^{n-1}, \quad (13)$$

where $\eta,\eta_0,n-$ are parameters of the hysteresis loop; $\Gamma\left(\frac{n+3}{2}\right)\wedge B(n,n)$ are alpha and beta functions, respectively.

For the case when **n=2**, based on expressions (12), we write the linearization coefficients according to Davidenko's hypothesis as follows:

$$\eta_1=\frac{\eta\Gamma\left(\frac{5}{2}\right)}{3\pi B(2,2)}2^{\frac{1}{2}}\sigma_{x_1}=\frac{\eta\frac{4!\sqrt{\pi}}{4^2 2!}}{3\pi\frac{1!\,1!}{3!}}2^{\frac{1}{2}}\sigma_{x_1}=\frac{\frac{3}{4}\sqrt{\pi}\eta}{3\pi\cdot\frac{1}{6}}2^{\frac{1}{2}}\sigma_{x_1}=C_{11}\sigma_{x_1};$$
$$\eta_2=\frac{\eta\Gamma\left(\frac{5}{2}\right)}{6\pi}2^{\frac{7}{2}}\sigma_{x_1}=\frac{\eta\frac{4!\sqrt{\pi}}{4^2 2!}}{6\pi}2^{\frac{7}{2}}\sigma_{x_1}=\frac{\frac{3}{4}\sqrt{\pi}\eta}{6\pi}2^{\frac{7}{2}}\sigma_{x_1}=C_{21}\sigma_{x_1};$$
$$\nu_1=\frac{\eta_0\Gamma\left(\frac{5}{2}\right)}{3\pi B(2,2)}2^{\frac{1}{2}}\sigma_{x_2}=\frac{\eta_0\frac{4!\sqrt{\pi}}{4^2 2!}}{3\pi\frac{1!\,1!}{3!}}2^{\frac{1}{2}}\sigma_{x_2}=\frac{\frac{3}{4}\sqrt{\pi}\eta_0}{3\pi\frac{1}{6}}2^{\frac{1}{2}}\sigma_{x_2}=\mathrm{C}_{31}\sigma_{x_2};$$
$$\nu_2=\frac{\eta_0\Gamma\left(\frac{5}{2}\right)}{6\pi}2^{\frac{7}{2}}\sigma_{x_2}=\frac{\eta_0\frac{4!\sqrt{\pi}}{4^2 2!}}{6\pi}2^{\frac{7}{2}}\sigma_{x_2}=\frac{\frac{3}{4}\sqrt{\pi}\eta_0}{6\pi}2^{\frac{7}{2}}\sigma_{x_2}=\mathrm{C}_{41}\sigma_{x_2}, \quad (14)$$

where

$$C_{11}=1.196826491\eta;C_{21}=0.7978843277\eta;$$
$$\mathrm{C}_{31}=1.196826491\eta_0;\mathrm{C}_{41}=0.7978843277\eta_0.$$

The expressions (12), together with the linearization coefficients (14), represent bivariate functions with respect to the root-mean-square valuesσ_{x_1},$\wedge\,\sigma_{x_2}$ They allow evaluating the dynamics of random parametric vibrations in a mechanical system protected against damped oscillations for various parameter values.

If we accept the linearization coefficients (14) for expressions (12), then we obtain the following:

$$\sum_{k=0}^{8} H_{1k}\sigma_{x_2}^k = 0; \quad \sum_{k=0}^{8} H_{2k}\sigma_{x_2}^k = 0, \tag{15}$$

In this case, the coefficients $H_{1k}$and $H_{2k}$are determined depending on both the system parameters from expressions (12) and the root-mean-square value σ_{x_1},$\wedge$ σ_{x_2}.

For two polynomials in system (15) to be equal, their coefficients must be $H_{1k} = H_{2k}$

First, considering the higher-order nonlinearity, i.e., when $k = 8$, we equate the coefficients of the polynomials (15) before the term $\sigma_{x_2}^8$

$$H_{18} = H_{28} = 262144\omega^8\omega_2^{16}C_{31}^2C_{41}^2(C_{31}^2 + C_{41}^2)^2(1+\mu)^8$$

It can be seen that the difference $H_{18} - H_{28} = 0$ is exact. When$k = 7$, we equate the coefficients of the polynomials (15) before the term $\sigma_{x_2}^7$. In this case, the following expression can be derived from the equality

$$H_{17} - H_{27} = 0:$$
$$\sigma_{x_1}\left(\mu + \frac{1}{3}\right)q\omega C_{41}^2C_{21}(4\omega^2 + q^2)(C_{31}^2 - C_{41}^2) -$$
$$\frac{-1}{48}(\mu-1)\sigma_\xi^2\omega_1^2(8\omega^2 + q^2)(3C_{41}^4 + 4C_{41}^2C_{31}^2 + C_{31}^4) = 0. \tag{16}$$

This defined expression (16) also allows us to estimate the mean square value of σ_{x_1} with respect to the parameters explicitly participating in it. However, since the coefficient C_{11} and the frequency ω_2 are not explicitly participating in it, it does not allow us to analyze the change in the mean square value of σ_{x_1} with respect to this coefficient and frequency. Taking this into account, we equate the coefficients of the polynomials (15) in front of the term $\sigma_{x_2}^6$ when $k = 6$. In this case, the following expression can be obtained from the equality $H_{16} - H_{26} = 0$:

$$H_2\sigma_{x_1}^2 + H_1\sigma_{x_1} + H_0 = 0, \tag{17}$$

Where

$$H_2 = \frac{-(3\mu+1)^2}{6}(2C_{11}C_{31}^2C_{41} - C_{11}C_{41}^3 + C_{21}C_{31}^3 - C_{21}C_{31}C_{41}^2)C_{21}C_{41}^2\omega_1^2 \times$$
$$\times (4\omega^2 + q^2)^2q\omega^2;$$
$$H_1 = \{$$
$$\times q\omega\omega_1^2 - \frac{1}{2}\omega_2^2\left(\mu + \frac{1}{3}\right)C_{21}(4\omega^2 + q^2)q(\mu+1)^2\omega \times$$
$$\times C_{41}^5 - \frac{1}{192}\omega_1^4\left((-104\mu^2 + 112\mu + 56)\omega^2 + q^2(11\mu^2 - 2\mu - 1)\right) \times$$
$$\times C_{21}\sigma_\xi^2C_{31}C_{41}^4 + C_{31}^2$$
$$\times C_{21}(4\omega^2 + q^2)q\omega\omega_1^2 + \omega_2^2\left(\mu + \frac{1}{3}\right)C_{21}(4\omega^2 + q^2)q(1+\mu)^2\omega C_{41}^3 +$$
$$\frac{+1}{48}\omega_1^4C_{21}\sigma_\xi^2\left((112\mu^2 - 64\mu - 32)\omega^2 + q^2(17\mu^2 - 10\mu - 5)\right)C_{31}^2C_{41}^2 +$$
$$\frac{+1}{96}\omega_1^4\sigma_\xi^2(8\omega^2 + q^2)C_{31}^4(7\mu^2 - 6\mu - 3)C_{11}C_{41} + \frac{1}{64}\omega_1^4\left(\mu + \frac{1}{3}\right)C_{21}\sigma_\xi^2 \times$$
$$\times (8\omega^2 + q^2)C_{31}^5(\mu-1)\}(4\omega^2 + q^2)\omega;$$
$$H_0 = \frac{-\sigma_\xi^2}{32}\{(4\omega^2 + q^2)\left(\mu^3\omega_2^2 + \left(\frac{7}{3}\omega_1^2 + \omega_2^2\right)\mu^2 - (2\omega_1^2 + \omega_2^2)\mu - \omega_1^2 - \omega_2^2\right) \times$$
$$\times \omega C_{41}^5 + \frac{5}{16}\sigma_\xi^2\left(\mu^2 - \frac{22}{15}\mu + 1\right)\omega_1^4qC_{31}C_{41}^4 + \frac{8}{3}(4\omega^2 + q^2) \times$$
$$\times \left(\mu^3\omega_2^2 + \left(\frac{3}{2}\omega_1^2 + \omega_2^2\right)\mu^2 + (-2\omega_1^2 - \omega_2^2)\mu - \omega_1^2 - \omega_2^2\right)\omega C_{31}^2C_{41}^3 +$$
$$\frac{+1}{4}\sigma_\xi^2\left(\mu^2 - \frac{8}{3}\mu + 1\right)\omega_1^4qC_{31}^3C_{41}^2 + (4\omega^2 + q^2) \times$$
$$\times \left(\mu^3\omega_2^2 + \left(\frac{7}{3}\omega_1^2 + \omega_2^2\right)\mu^2 + (-2\omega_1^2 - \omega_2^2)\mu - \omega_1^2 - \omega_2^2\right)\omega C_{31}^4C_{41} +$$
$$\frac{+1}{48}qC_{31}^5\omega_1^4\sigma_\xi^2(\mu-1)^2\}(8\omega^2 + q^2)\omega_1^2.$$

From the quadratic equation (17)

$$\sigma_{x_1} = \frac{-H_1 \pm \sqrt{H_1^2 - 4H_2H_0}}{2H_2}. (18)$$

In the root-mean-square expression (18), we divide both the numerator and denominator of the fraction by $q^9$and write it as follows:

$$\sigma_{x_1} = -H_{1*\pm}\sqrt{H_{1*}{}^2 - 4\frac{H_{2*}H_{0*}}{2H_{2*},(19)}}$$

where

$$H_{2*=}\frac{-(3\mu+1)^2}{6}(2C_{11}C_{31}^2C_{41} - C_{11}C_{41}^3 + C_{21}C_{31}^3 - C_{21}C_{31}C_{41}^2)\times$$
$$\times C_{21}C_{41}^2\Omega_1^2(4\Omega_0^2+1)^2\Omega_0^2;$$
$$H_{1*=}$$
$$\times \Omega_1^2 - \frac{1}{2}\Omega_2^2\left(\mu+\frac{1}{3}\right)C_{21}(4\Omega_0^2+1)(\mu+1)^2\Omega_0 \times$$
$$\times C_{41}^5 - \frac{1}{192}\Omega_1^4\left((-104\mu^2+112\mu+56)\Omega_0^2 + (11\mu^2-2\mu-1)\right)\times$$
$$\times C_{21}\sigma_\xi^2C_{31}C_{41}^4 + C_{31}^2$$
$$\times C_{21}(4\Omega_0^2+1)\Omega_0\ \Omega_1^2 + \Omega_2^2\left(\mu+\frac{1}{3}\right)C_{21}(4\Omega_0^2+1)(1+\mu)^2\Omega_0\ C_{41}^3 +$$
$$\frac{+1}{48}\Omega_1^4C_{21}\sigma_\xi^2\left((112\mu^2-64\mu-32)\Omega_0^2 + (17\mu^2-10\mu-5)\right)C_{31}^2C_{41}^2 +$$
$$\frac{+1}{96}\Omega_1^4\sigma_\xi^2(8\Omega_0^2+1)C_{31}^4(7\mu^2-6\mu-3)C_{11}C_{41} + \frac{1}{64}\Omega_1^4\left(\mu+\frac{1}{3}\right)C_{21}\sigma_\xi^2 \times$$
$$\times (8\Omega_0^2+1)C_{31}^5(\mu-1)\}(4\Omega_0^2+1)\Omega_0\ ;$$
$$H_0 = \frac{-1}{32}\sigma_\xi^2\{(4\Omega_0^2+1)$$
$$-\Omega_2^2\Omega_0\ C_{41}^5 + \frac{5}{16}\sigma_\xi^2\left(\mu^2 - \frac{22}{15}\mu + 1\right)\Omega_1^4C_{31}C_{41}^4 + \frac{8}{3}(4\Omega_0^2+1)\times$$
$$\times\left(\mu^3\Omega_2^2 + \left(\frac{3}{2}\Omega_1^2+\Omega_2^2\right)\mu^2 + (-2\Omega_1^2-\Omega_2^2)\mu - \Omega_1^2 - \Omega_2^2\right)\Omega_0\ C_{31}^2C_{41}^3 +$$
$$\frac{+1}{4}\sigma_\xi^2\left(\mu^2 - \frac{8}{3}\mu + 1\right)\Omega_1^4C_{31}^3C_{41}^2 + (4\Omega_0^2+1)\times$$
$$\times\left(\mu^3\Omega_2^2 + \left(\frac{7}{3}\Omega_1^2+\Omega_2^2\right)\mu^2 + (-2\Omega_1^2-\Omega_2^2)\mu - \Omega_1^2 - \Omega_2^2\right)\Omega_0\ C_{31}^4C_{41} +$$
$$\frac{+1}{48}C_{31}^5\Omega_1^4\sigma_\xi^2(\mu-1)^2\}(8\Omega_0^2+1)\Omega_1^2;$$
$$\Omega_0 = \frac{\omega}{q}; \Omega_1 = \frac{\omega_1}{q}; \Omega_2 = \frac{\omega_2}{q}.$$

Conclusion. A method for analytically determining the root mean square values of random parametric oscillations of a mechanical system protected from vibrations using the Ito method has been developed. The obtained expression for the root mean square values (19) allows a complete analysis of the dynamics of random parametric oscillations of a mechanical system protected from vibrations with an elastic dissipative characteristic of the hysteresis type at different values of the system parameters and an assessment of the effectiveness of the dynamic damper.

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