ISNN:2181-1296



SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ ШАРОФА РАШИДОВА

SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

ILMIY AXBOROTNOMA

НАУЧНЫЙ ВЕСТНИК
SCIENTIFIC JOURNAL

ANIQ VA TABIIY FANLAR SERIYASI

Matematika, Mexanika, Informatika
Fizika, Kimyo, Biologiya, Geografiya

№ 1/1 (143)
2024

ISSN 2181-1296

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2024-yil, 1-son (143/1) **ANIQ VA TABIY FANLAR SERIYASI**

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Samarqand viloyat matbuot boshqarmasida ro‘yxatdan o‘tish tartibi 09-25.
Jurnal 1999-yildan chop qilina boshlagan va OAK ro‘yxatiga kiritilgan.

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Obuna indeksi – yakka tartibdagi obunachilar uchun **- 5583,**
tashkilot, korxonalar uchun **- 5584**

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UDK: 517.9

QUALITATIVE PROPERTIES FOR COMPLEX BIOLOGICAL POPULATION PROCESSES

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Kelib tushgan: 04.02.2024
Ko‘rib chiqildi: 24.02.2024
Qabul qilindi: 27.02.2024
Nashr qilindi: 18.03.2024

Nashriyotchi: Sharof Rashidov nomidagi Samarqand davlat universiteti.

Abstract. In this work, we study the qualitative properties for complex biological population processes; on its basis, we solve the problem of choosing an initial approximation for the iterative process, which leads to rapid convergence to the solution of the Cauchy problem, depending on the values of the numerical parameters and the initial data. For this purpose, an asymptotic representation of the solution was used as an initial approximation, which made it possible to conduct a numerical experiment and visualize the process described by a system of quasilinear parabolic equations, depending on the values included in the system of numerical parameters. The properties of solutions to the problem in the case of double nonlinearity are studied. The main research method is the self-similar approach.

Keywords: Cauchy problem, slow diffusion, fast diffusion, self-similar equations of systems, biological population.

Murakkab holatdagi biologik populyatsiya jarayonlari uchun sifat xossalari

Annotatsiya. Ushbu maqolada murakkab holatdagi biologik populyatsiya jarayonlari uchun sifat xossalari o‘rganilgan, uning asosida iterativ jarayon uchun boshlang‘ich yaqinlashuvni tanlash masalasi ko‘rsatilgan, bu Koshi masalasining yechimiga tez yaqinlashishiga olib keladi va chekli ayirmali usullarni qo‘llash uchun ishlatiladi. Buning uchun dastlabki yaqinlashish sifatida yechimning asimptotik holati ko‘rsatilgan, bu sonli hisoblash jarayonini o‘tkazish va spnli parametrlar tizimiga kiritilgan qiymatlarga qarab kvazichiziqli parabolik tenglamalar sistemasi tomonidan tasvirlangan jarayonni tasavvur qilish imkonini beradi. Masalani yechishning xossalari ikki karra nochiziqli holatida tekshiriladi. Asosiy tadqiqot usuli – avtomodel tenglamalar usuli.

Kalit so‘zlar: Koshi masalasi, tez diffuziya, sekin diffuziya, avtomodel tenglamalar sistemasi, biologik populyatsiya.

Качественные свойства сложных биологических популяционных процессов

Аннотация. В этой работе мы изучаем качественные свойства сложных биологических популяционных процессов; на его основе решается задача выбора начального приближения для итерационного процесса, что приводит к быстрой сходимости к решению задачи Коши в зависимости от значений числовых параметров и начальных данных. Для этого в качестве начального приближения использовалось асимптотическое представление решения, что позволило провести численный эксперимент и визуализировать процесс, описываемый системой квазилинейных параболических уравнений, в зависимости от значений, входящих в систему числовых параметров. Изучены свойства решений задачи в случае двойной нелинейности. Основным методом исследования является автомодельный подход.

Ключевые слова: Задача Коши, медленная диффузия, быстрая диффузия, автомодельные уравнения систем, биологическая популяция.

Consider in the area $Q=\{(t,x): t\in R_+, x\in R\}$ parabolic system of two quasilinear reaction-diffusion equations

$$\begin{cases}\dfrac{\partial u_1}{\partial t}=\nabla\left(u_1^{m_1-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_1\right)+a_1(t)u_1-b_1(t)u_2^{\beta_1},\\ \dfrac{\partial u_2}{\partial t}=\nabla\left(u_2^{m_2-1}\left|\nabla u_2^k\right|^{p-2}\nabla u_2\right)+a_2(t)u_2-b_2(t)u_1^{\beta_2},\end{cases} \tag{1}$$

$$u_1\big|_{t=0}=u_{10}(x),\quad u_2\big|_{t=0}=u_{20}(x), \tag{2}$$

which describes the process of a biological population of the Kolmogorov-Fisher type in a nonlinear two-component medium, the coefficients of mutual diffusion of which are, respectively, equal $u_1^{m_1-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_1$, $u_2^{m_2-1}\left|\nabla u_2^k\right|^{p-2}\nabla u_2$. Numeric parameters $m_1,m_2,n,p,\beta_1,\beta_2$ - positive real numbers, $\nabla(.)-\underset{x}{grad}(.)$, $\beta_1,\beta_2\geq 1$, $x\in R^N$; $u_1=u_1(t,x)\geq 0$, $u_2=u_2(t,x)\geq 0$ - desired solutions [2].

We will study the properties of solutions to problem (1), (2) based on a self-similar analysis of solutions to the system of equations constructed by the method of nonlinear splitting and reference equations [4].

Note that the replacement in (1)

$$u_1(t,x)=e^{-\int a_1(t)dt}v_1(\tau(t),x),\quad u_2(t,x)=e^{-\int a_2(t)dt}v_2(\tau(t),x)$$

lead it to the form:

$$\begin{cases}\dfrac{\partial v_1}{\partial \tau}=\nabla\left(v_1^{m_1-1}\left|\nabla v_1^k\right|^{p-2}\nabla v_1\right)+b_1(t)e^{\beta_1\int a_2(t)dt-[(p-2)k+m_1]\int a_1(t)dt}v_2^{\beta_1},\\ \dfrac{\partial v_2}{\partial \tau}=\nabla\left(v_2^{m_2-1}\left|\nabla v_2^k\right|^{p-2}\nabla v_2\right)+b_2(t)e^{\beta_2\int a_1(t)dt-[(p-2)k+m_2]\int a_2(t)dt}v_1^{\beta_2}.\end{cases} \tag{3}$$

$$v_1\big|_{t=0}=v_{10}(x),\quad v_2\big|_{t=0}=v_{20}(x). \tag{4}$$

If $[(p-2)k+(m_1-1)]\int a_1(t)dt=[(p-2)k+(m_2-1)]\int a_2(t)dt$, then by choosing

$$\tau(t)=\frac{e^{[(m_1-1)+(p-2)k]\int a_1(t)dt}}{[(m_1-1)+(p-2)k]\int a_1(t)dt}=\frac{e^{[(m_2-1)+(p-2)k]\int a_2(t)dt}}{[(m_2-1)+(p-2)k]\int a_2(t)dt},$$

We obtain the following system of equations:

$$\begin{cases}\dfrac{\partial v_1}{\partial \tau}=\nabla\left(v_1^{m_1-1}\left|\nabla v_1^k\right|^{p-2}\nabla v_1\right)+k_1(t)\tau^{l_1}v_2^{\beta_1},\\ \dfrac{\partial v_2}{\partial \tau}=\nabla\left(v_2^{m_2-1}\left|\nabla v_2^k\right|^{p-2}\nabla v_2\right)+k_2(t)\tau^{l_2}v_1^{\beta_2},\end{cases} \tag{5}$$

where $k_1(t)=b_1(t)\left([(m_1-1)+(p-2)k]\int a_1(t)dt\right)^{l_1}$, $\quad l_1=\dfrac{\beta_1\int a_2(t)dt-[m_1+(p-2)k]\int a_1(t)dt}{[m_1+(p-2)k]\int a_1(t)dt}$

$k_2(t)=b_2(t)\left([(m_2-1)+(p-2)k]\int a_2(t)dt\right)^{l_2}$, $\quad l_2=\dfrac{\beta_2\int a_1(t)dt-[m_2+(p-2)k]\int a_2(t)dt}{[m_2+(p-2)k]\int a_2(t)dt}$

If $l_i=0$ and $k_i(t)=const$, $i=1,2$, then the system has the form:

$$\begin{cases}\dfrac{\partial v_1}{\partial \tau}=\nabla\left(v_1^{m_1-1}\left|\nabla v_1^k\right|^{p-2}\nabla v_1\right)+k_1v_2^{\beta_1},\\ \dfrac{\partial v_2}{\partial \tau}=\nabla\left(v_2^{m_2-1}\left|\nabla v_2^k\right|^{p-2}\nabla v_2\right)+k_2v_1^{\beta_2},\end{cases} \tag{6}$$

Below we describe one of the ways to obtain a self-similar system for the system of equations (5). It consists of the following. Let us first find the solution to the system of ordinary differential equations

$$\begin{cases}\dfrac{d\bar{v}_1}{d\tau}=k_1(t)\tau^{l_1}v_2^{\beta_1},\\ \dfrac{d\bar{v}_2}{d\tau}=k_2(t)\tau^{l_2}v_1^{\beta_2},\end{cases}$$

In the form

$$\bar{v}_1(\tau)=(\tau(t))^{-\alpha_1},\ \alpha_1=\frac{1+\beta_1}{\beta_1\beta_2-1},\ \ \bar{v}_2(\tau)=(\tau(t))^{-\alpha_2},\ \alpha_2=\frac{1+\beta_2}{\beta_1\beta_2-1},$$

for the case $l_i=0$, and $k_i(t)=const$, $i=1,2$. And in the case $l_i\neq 0$, and $k_i(t)=const$, $i=1,2$ find a solution to the system of ordinary differential equations

$$\begin{cases}\dfrac{d\bar{v}_1}{d\tau}=k_1\tau^{l_1}v_2^{\beta_1},\\ \dfrac{d\bar{v}_2}{d\tau}=k_2\tau^{l_2}v_1^{\beta_2},\end{cases}$$

in the form

$$\bar{v}_1(\tau)=(\tau(t))^{-\alpha_1},\ \alpha_1=\frac{(l_1+1)+\beta_1(l_2+1)}{\beta_1\beta_2-1},\ \ \bar{v}_2(\tau)=(\tau(t))^{-\alpha_2},\ \alpha_2=\frac{(l_2+1)+\beta_2(l_1+1)}{\beta_1\beta_2-1}\ .$$

Then solution of the system (5) is sought in the form

$$\begin{aligned}v_1(t,x)&=\bar{v}_1(\tau)w_1(\tau(t),x),\\ v_2(t,x)&=\bar{v}_2(\tau)w_2(\tau(t),x),\end{aligned}\qquad(7)$$

and $\tau=\tau(t)$ choose as

$$\tau_1(\tau)=\int_0^{\tau}\bar{v}_1^{(p-2)k+(m_1-1)}(t)dt=\begin{cases}\dfrac{(T+\tau)^{1-\alpha_1[(p-2)k+(m_1-1)]}}{1-\alpha_1[(p-2)k+(m_1-1)]}, & if\ \ 1-\alpha_1[(p-2)k+(m_1-1)]\neq 0,\\ \ln(T+\tau), & if\ \ 1-\alpha_1[(p-2)k+(m_1-1)]=0,\\ (T+\tau), & if\ \ p=2\ \ u\ \ m_1=1,\end{cases}$$

if $\alpha_1[(p-2)k+(m_1-1)]=\alpha_2[(p-2)k+(m_2-1)]$.

Then for $w_i(\tau(t),x),\ i=1,2$ we get the following system of equation:

$$\begin{cases}\dfrac{\partial w_1}{\partial\tau}=\nabla\left(w_1^{m_1-1}\left|\nabla w_1^k\right|^{p-2}\nabla w_1\right)+\psi_1(-w_1+w_2^{\beta_1}),\\ \dfrac{\partial w_2}{\partial\tau}=\nabla\left(w_2^{m_2-1}\left|\nabla w_2^k\right|^{p-2}\nabla w_2\right)+\psi_2(-w_2+w_1^{\beta_2}),\end{cases}\qquad(8)$$

where

$$\psi_1=\begin{cases}\dfrac{1}{(1-\alpha_1[(p-2)k+(m_1-1)])\tau}, & and\ \ 1-\alpha_1[(p-2)k+(m_1-1)]>0,\\ \alpha_1 c_1^{-\left(1-\alpha_1[(p-2)k+(m_1-1)]\right)}, & and\ \ 1-\alpha_1[(p-2)k+(m_1-1)]=0,\end{cases}$$

$$\psi_2=\begin{cases}\dfrac{1}{(1-\alpha_2[(p-2)k+(m_2-1)])\tau}, & and\ \ 1-\alpha_2[(p-2)k+(m_2-1)]>0,\\ \alpha_2 c_2^{-\left(1-\alpha_2[(p-2)k+(m_2-1)]\right)}, & and\ \ 1-\alpha_2[(p-2)k+(m_2-1)]=0.\end{cases}$$

If $1-\alpha_1[(p-2)k+(m_1-1)]=0$, self-similar solution of system (8) has the form

$$w_i(\tau(t),x)=f_i(\xi),\ i=1,2,\ \xi=x/[\tau(t)]^{1/p}.\qquad(9)$$

Then substituting (9) into (8) with respect to $f_i(\xi)$ we obtain the system of self-similar equations

$$\begin{cases}\xi^{1-N}\dfrac{d}{d\xi}\left(\xi^{N-1}f_1^{m_1-1}\left|\dfrac{df_1^k}{d\xi}\right|^{p-2}\dfrac{df_1}{d\xi}\right)+\dfrac{\xi}{p}\dfrac{df_1}{d\xi}+\mu_1(-f_1+f_2^{\beta_1})=0,\\ \xi^{1-N}\dfrac{d}{d\xi}\left(\xi^{N-1}f_2^{m_2-1}\left|\dfrac{df_2^k}{d\xi}\right|^{p-2}\dfrac{df_2}{d\xi}\right)+\dfrac{\xi}{p}\dfrac{df_2}{d\xi}+\mu_2(-f_2+f_1^{\beta_2})=0.\end{cases}\qquad(10)$$

where $\mu_1=\dfrac{1}{(1-\alpha_1[(p-2)k+(m_1-1)])}$ and $\mu_2=\dfrac{1}{(1-\alpha_2[(p-2)k+(m_2-1)])}$.

System (10) has an approximate solution of the form

$$\bar{f}_1 = A(a-\xi^{\gamma})_+^{n_1},\ \gamma = p/(p-1),\ \ \bar{f}_2 = B(a-\xi^{\gamma})_+^{n_2},$$

where *A and B* constant and

$$n_1 = \frac{k(p-2)+1}{(m_1-1)+k(p-2)},\ n_2 = \frac{k(p-2)+1}{(m_2-1)+k(p-2)}.$$

In this paper, on the basis of the above methodology, the qualitative properties of solutions of system (1) are investigated, on its basis the problem of choosing an initial approximation for an iterative one is solved, which leads to fast convergence to the solution of the Cauchy problem (1), (2), depending on the value of the numerical parameters and initial data. For this purpose, the asymptotic representation of the solution found by us was used as an initial approximation [1].

This made it possible to perform a numerical experiment and visualize the process described by system (1), depending on the values included in the system of numerical parameters [2].

Investigation of the qualitative properties of system (1) made it possible to carry out a numerical experiment depending on the values included in the system of numerical parameters. For this purpose, the asymptotic of the solutions were used as an initial approximation.

Case $n_1 > 0, n_2 > 0, q_1 > 0, q_2 > 0$ (slow diffusion). Applying the method [1] for solving equation (10), we obtain the following functions

$$\bar{\theta}_1(\xi) = (a-\xi)_+^{n_1},\ \bar{\theta}_2(\xi) = (a-\xi\ )_+^{n_2},$$

Finite solution of the system (10) at $\xi \to a_-$ has an asymptotic $f_i(\xi) \sim \vartheta_i(\xi)$.

Case $n_1 > 0, n_2 > 0, q_1 < 0, q_2 < 0$ (fast diffusion). For (10) we have

$$\chi_1(\xi) = (a+\xi\ )^{n_1},\ \chi_2(\xi) = (a+\xi\ )^{n_2},$$

where $a > 0$, $q_1 = (m_1-1)+k(p-2)$, $q_2 = (m_2-1)+k(p-2)$.

At $\xi \to +\infty$ solution of problem (10) vanishing at infinity has the asymptotic $f_i(\xi) \sim \chi_i(\xi),\ \ i = 1,2$.

Thus, proposed nonlinear mathematical model of a biological population with double nonlinearity correctly reflects the process under study. Analyzing the results based on the resulting estimates of decisions gives a comprehensive picture of the process.

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \nabla\left(u_1^{m_1-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_1\right) + a_1(t)u_1 - b_2(t)u_1^{\beta_1}, \\ \dfrac{\partial u_2}{\partial t} = \nabla\left(u_2^{m_2-1}\left|\nabla u_2^k\right|^{p-2}\nabla u_2\right) + a_2(t)u_2 - b_2(t)u_1^{\beta_2}. \end{cases}$$

Results of numerical experiments and visualization

When solving the problem numerically, the equation is approximated on a grid using an implicit scheme of variable directions (for the multidimensional case) in combination with the balance method. Iterative processes were built on the basis of the Picard, Newton method as well as special method.

A computational algorithm has been developed. When developing software that illustrates the simulation process of solution behavior over time (visualization), the Python 3.12 environment was used with the inclusion of the Open GL graphics library.

The results of computational experiments show that all iterative methods are suitable for the constructed scheme. To achieve the same accuracy, the Newton method requires fewer iterations than the Picard method. In some cases, the number of iterations is almost twice, and the maximum number of iterations is 3-4 times less than in other methods. Since power-law nonlinearity is present in the right-hand side of equation (1), naturally the special method gives better results than the Picard method.

As a test example, we used solutions of equation (1) obtained by the methods of reference equations and nonlinear splitting [2]. Figures 1-4 show the calculation results for various values of parameters $p, n, m, k, l, \mu, \beta$ and time.

The results of a computational experiment for various values of the parameters included in the equation in the two-dimensional case are given below:

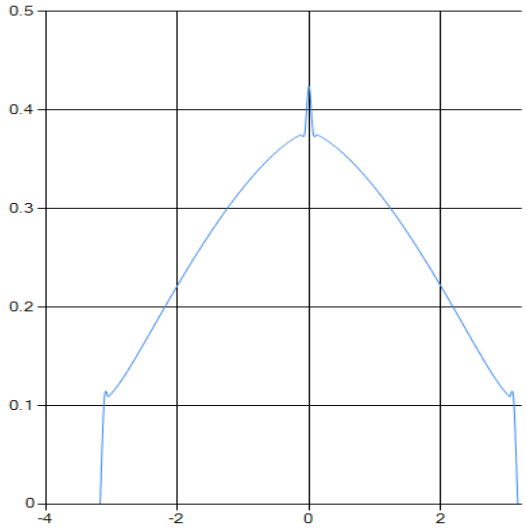

Fig.1. *p=2.5, n=1, m=1.4, k=1, l=1*

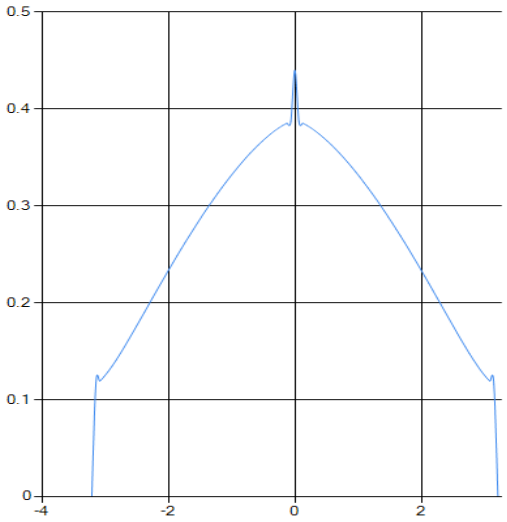

Fig.2. *p=2.4, n=1, m=1.2, k=1.3, l=1*

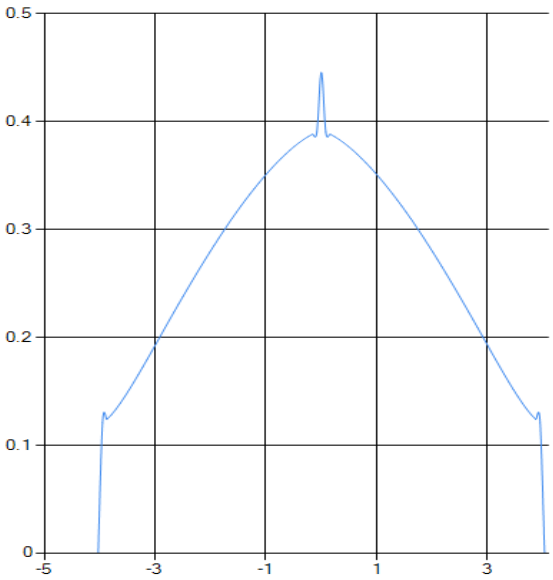

Fig.3. *p=2.1, n=1.2, m=1, k=1, l=1*

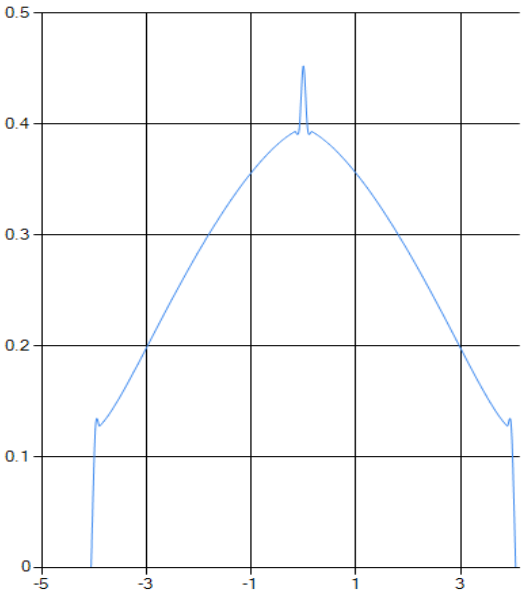

Fig.4. *p=2.3, n=1, m=1.2, k=1.3, l=1*

Conclusion

The results of the search for the solution of the given problem (1)-(2) show that it is necessary to analyze and find self-similar or approximately self-similar solutions for the study of nonlinear heat diffusion and nonlinear thermal conductivity processes. For this purpose, it is necessary to observe the qualitative properties of systems of equations, as well as cases of compact asymptotic support of non divergent solutions of systems of equations. New effects related to the problem were discovered in the search for solutions of self-similar models. From finite-difference schemes, it is clear that a nonlinear decomposition approach and a similar method can be developed in practice. In addition, the excellent results of creating iterative approaches using Newton's and Picard's methods in finding numerical solutions to systems of nonlinear standard equations lead to numerically computable solutions. Using the values of the specified parameters, it is possible to see the effect of the speed of heat propagation and the conditions of spatial localization.

Acknowledgements

We are very grateful to the organizing committee for the helpful suggestions for the development of this research work and will further improve the research work.

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ISSN 2181-1296

