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THE NUMBER OF EIGENVALUES OF THE DISCRETE SCHRÖDINGER OPERATOR

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Abstract. In this paper we consider the family $H_{\gamma\lambda\mu}$ of Schrödinger operators, associated to the Bose-Hubbard Hamiltonian of a single boson on the one dimensional lattice. We find suffiently conditions for the parameters, such that the operator $H_{\gamma\lambda\mu}$ has no one eigenvalues below the essential spectrum and three eigenvalues above the essential spectrum.

Keywords: Schrödinger operator, essential spectrum, one-dimensional lattice.

Diskret Shredinger operatori xos qiymatlari soni

Annotatsiya. Ushbu maqolada bir o‘lchamli panjaradagi bir bozonli Bose-Hubbard Gamiltonianiga mos $H_{\gamma\lambda\mu}$ Shredinger operatorlari oilasini qaraymiz. $H_{\gamma\lambda\mu}$ operator muhim spektrdan chapda birorta ham xos qiymatga ega bo’lmasligi va muhim spektrdan o’ngda uchta xos qiymatga ega bo’lishi uchun yetarli shart ko’rsatiladi.

Kalit so’zlar: Shredinger operatori, muhim spektr, bir o’lchamli panjara.

Число собственных значений дискретного оператора Шредингера

Аннотация. В этой статье мы рассматриваем семейство $H_{\gamma\lambda\mu}$ операторов Шредингера, связанных с гамильтонианом Бозе-Хаббарда одиночного бозона в одномерной решетке Z. Мы находим достаточные условия на параметров, при которых оператор $H_{\gamma\lambda\mu}$ не имеет собственных значений ниже существенного спектра и имель три собственных значений више существенного спектра.

Ключевые слова: оператор Шредингера, существенный спектр, одномерная решетка.

Introduction. In this paper we consider a one-particle discrete Schrödinger operator, acting in Hilbert space of even functions $l^{2,e}(Z)$ as

$$\widehat{H}_{\gamma\lambda\mu} = \widehat{H}_0 + \widehat{V}_{\gamma\lambda\mu},$$

where $\widehat{H}_0$ is the discrete Laplacian and $\widehat{V}_{\gamma\lambda\mu}$ is a multiplication operator by the function $\hat{v}_{\gamma\lambda\mu}$ of the form

$$\hat{v}_{\gamma\lambda\mu}(s) = \begin{cases} \gamma, & |s| = 0 \\ \frac{\lambda}{2}, & |s| = 1 \\ \frac{\mu}{2}, & |s| = 2 \\ 0, & |s| > 2, \end{cases}$$

with $\gamma, \lambda, \mu \in \mathbb{R}$.

First we observe that the essential spectrum of $\widehat{H}_{\gamma\lambda\mu}$ consists of a segment $[0,2]$.

To study the number of eigenvalues of $\widehat{H}_{\gamma\lambda\mu}$ we introduce the Fredholm determinant $\Delta_{\gamma\lambda\mu}(z)$, associated to $\widehat{H}_{\gamma\lambda\mu}$ in $l^{2,e}(Z)$. It is well-known that the eigenvalues of $\widehat{H}_{\gamma\lambda\mu}$ are zeros of this determinant, and vice versa (see Lemma 1).

We are interested in the number of eigenvalues of $\widehat{H}_{\gamma\lambda\mu}$ and their arrangement depending on the parameters $\gamma, \lambda$and μ. For this purpose, we find the function $C(\gamma,\lambda,\mu)$ which appear as constants in front of the main term in the asymptotics of the Fredholm determinant $\Delta_{\gamma\lambda\mu}(z)$, as z approaches to the top of the essential spectrum (Lemma 4).

Next, we apply this fact to find simple conditions for the potential under which the discrete Schrödinger operator $\widehat{H}_{\gamma\lambda\mu}$ has three eigenvalues above the essential spectrum.

The problem of determining the number and location of eigenvalues of the discrete Schrödinger operator in a one-dimensional case was studied in [1] and [3].

In [1] for the discrete Schrödinger operator $\widehat{H}_{\gamma\lambda 0}$, the presence of one or two eigenvalues, situated as below the bottom of the essential spectrum, as well as above its top was proved. Moreover, it was established that this operator can have two eigenvalues outside of the essential spectrum such that one of them is situated below the bottom of the essential spectrum, and other one above its top.

In [3] the two-particle Schrödinger operators $\widehat{H}_{\gamma\lambda 0}(K)$ with the fixed quasi-momentum of particles pair $K \in T$ associated to the Fermi-Hubbard Hamiltonian $\mathbb{H}_{\gamma\lambda 0}(K)$ of a system of two identical quantum-mechanical particles (fermions) interacting via zero-range potential $\gamma \in \mathbb{R}$ on site and potential $\lambda \in \mathbb{R}$, on neighboring sites was studied.

The one-particle Hamiltonian. The coordinate representation.

Let $\mathbb{Z}$ be the one-dimensional lattice and $\ell^{2,e}(\mathbb{Z})$ be the Hilbert space of square summable even functions on $\mathbb{Z}$. The one-particle discrete Schrödinger operator $\widehat{H}_{\gamma\lambda\mu}$, which are dependent on real parameters γ, λ and μ, acting in $\ell^{2,e}(\mathbb{Z})$ is given as

$$\widehat{H}_{\gamma\lambda\mu} = \widehat{H}_0 + \widehat{V}_{\gamma\lambda\mu},$$

where $\widehat{H}_0$ is the Teoplitz type operator

$$(\widehat{H}_0\hat{\varphi})(x) = \sum_{s\in Z} \hat{\varepsilon}\,(s)\hat{\varphi}(x+s),\ \hat{\varphi} \in \ell^{2,e}(\mathbb{Z})$$

and $\widehat{V}_{\gamma\lambda\mu}$ is a multiplication operator

$$(\widehat{V}_{\gamma\lambda\mu}\hat{\varphi})(x) := \hat{v}_{\gamma\lambda\mu}(x)\hat{\varphi}(x), \quad \hat{\varphi} \in \ell^{2,e}(\mathbb{Z}),$$

where, the functions $\hat{\varepsilon}(s)$ and $\hat{v}_{\gamma\lambda\mu}(s)$ are defined on $\mathbb{Z}$ as

$$\hat{\varepsilon}(s) = \begin{cases} 1, & |s| = 0 \\ -\dfrac{1}{2}, & |s| = 1 \\ 0, & |s| > 1 \end{cases} \quad \text{and} \quad \hat{v}_{\gamma\lambda\mu}(s) = \begin{cases} \gamma, & |s| = 0 \\ \dfrac{\lambda}{2}, & |s| = 1 \\ \dfrac{\mu}{2}, & |s| = 2 \\ 0, & |s| > 2, \end{cases}$$

respectively. We remark that $\widehat{H}_{\gamma\lambda\mu}$ is a bounded self-adjoint operator on $\ell^{2,e}(\mathbb{Z})$.

The momentum representation.

Let $\mathbb{T} = (-\pi;\pi]$ be a one-dimensional torus and $L^{2,e}(\mathbb{T}, dv)$ be the Hilbert space of square integrable even functions on $\mathbb{T}$, with dv being a (normalized) Haar measure on $\mathbb{T}$, $\left(dv(p) = \frac{dp}{2\pi}\right)$.

In the momentum representation, the operator $H_{\gamma\lambda\mu}$ acts on $L^{2,e}(\mathbb{T}, dv)$ as

$$H_{\gamma\lambda\mu} = H_0 + V_{\gamma\lambda\mu},$$

where H_0 is the multiplication operator by function $\varepsilon(p) = 1 - \cos p$

$$(H_0 f)(p) = \varepsilon(p) f(p), \quad f \in L^{2,e}(\mathbb{T}, dv),$$

and $V_{\gamma\lambda\mu}$ is the integral operator of rank at most three

$$(V_{\gamma\lambda\mu}f)(p)=\int_{\mathbb{T}}(\gamma+\mu\cos p\cos t+\lambda\cos 2p\cos 2t)f(t)dv(t),\quad f\in L^{2,e}(\mathbb{T},dv).$$

Spectral properties of the operators $H_{\gamma\lambda\mu}$

The perturbation operator $V_{\gamma\lambda\mu}$, $\gamma,\lambda,\mu\in\mathbb{R}$ is of rank at most 3, therefore by the well known Weyl's theorem the essential spectrum $\sigma_{ess}(H_{\gamma\lambda\mu})$ of $H_{\gamma\lambda\mu}$ does not depend on $\gamma,\lambda,\mu\in\mathbb{R}$ and coincides with the spectrum $\sigma(H_0)$ of H_0 (see), i.e., $\sigma_{ess}(H_{\gamma\lambda\mu})=\sigma(H_0)=[\min_{p\in\mathbb{T}}\varepsilon(p),\max_{p\in\mathbb{T}}\varepsilon(p)]=[0,2]$.

For any $\gamma,\lambda,\mu\in\mathbb{R}$, we introduce the Fredholm determinant $\Delta_{\gamma\lambda\mu}(z)$ associated to the Hamiltonian $H_{\gamma\lambda\mu}$ as

$$\Delta_{\gamma\lambda\mu}(z)=\begin{vmatrix}1+\gamma a(z) & \lambda b(z) & \mu c(z)\\ \gamma b(z) & 1+\lambda d(z) & \mu e(z)\\ \gamma c(z) & \lambda e(z) & 1+\mu f(z)\end{vmatrix},$$

where

$$a(z):=\int_{\mathbb{T}}\frac{dv(t)}{\varepsilon(t)-z},\qquad b(z):=\int_{\mathbb{T}}\frac{\cos t dv(t)}{\varepsilon(t)-z},$$

$$c(z):=\int_{\mathbb{T}}\frac{\cos 2t dv(t)}{\varepsilon(t)-z},\qquad d(z):=\int_{\mathbb{T}}\frac{\cos^2 t dv(t)}{\varepsilon(t)-z},$$

$$e(z):=\int_{\mathbb{T}}\frac{\cos t\cos 2t dv(t)}{\varepsilon(t)-z},\quad f(z):=\int_{\mathbb{T}}\frac{\cos^2 2t dv(t)}{\varepsilon(t)-z}$$

are regular functions in $z\in\mathbb{R}\backslash[0,2]$.

Lemma 1. For all $(\gamma,\lambda,\mu)\in\mathbb{R}^3$ the number $z\in\mathbb{R}\backslash[0,2]$ is an eigenvalue of the operator $H_{\gamma\lambda\mu}$ if and only if $\Delta_{\gamma\lambda\mu}(z)=0$.

Proof. Lemma 1 can be proved using Fredholm determinants theory.

Lemma 2. The functions $a(\cdot)$,$b(\cdot)$, $c(\cdot)$, $d(\cdot),e(\cdot)$ and $f(\cdot)$ are regular in $\mathbb{R}\backslash[0,2]$. Also, the following asymptotic relations are true for them:

$$a(z)=-\frac{1}{\sqrt{2}(z-2)^{\frac{1}{2}}}+O(-z)^{\frac{1}{2}},\ \text{as } z\to 2+,$$

$$d(z)=-\frac{1}{\sqrt{2}(z-2)^{\frac{1}{2}}}-1+O(-z)^{\frac{1}{2}},\ \text{as } z\to 2+,$$

$$f(z)=-\frac{1}{\sqrt{2}(z-2)^{\frac{1}{2}}}-2+O(-z)^{\frac{1}{2}},\ \text{as } z\to 2+.$$

$$b(z)=-\frac{1}{\sqrt{2}(z-2)^{\frac{1}{2}}}-1+O(-z)^{\frac{1}{2}},\ \text{as } z\to 2+,$$

$$c(z)=-\frac{1}{\sqrt{2}(z-2)^{\frac{1}{2}}}-2+O(-z)^{\frac{1}{2}},\ \text{as } z\to 2+,$$

$$e(z)=-\frac{1}{\sqrt{2}(z-2)^{\frac{1}{2}}}-2+O(-z)^{\frac{1}{2}},\ \text{as } z\to 2+.$$

Here the function $(z-2)^{\frac{1}{2}}$ denote those branch of analytic function that are positive for $z>2$.

Proof. The asymptotic relations of the above functions are calculated using residual theory.

Remark 3. Moreover, as the function under the integral sign are positive, monotonicity of the Lebesgue integral gives that the function $a(z)$ is negative too. Derivative of function $a(z)$ is negative, therefore this function monotonically increasing in the interval $(2,+\infty)$.

Lemma 4. The following asymptotics are true for the determinant $\Delta_{\gamma\lambda\mu}(z)$:

I. $\lim\limits_{z\to\pm\infty}\Delta_{\gamma\lambda\mu}(z)=1.$

II. $$\Delta_{\gamma\lambda\mu}(z)=\mathrm{C}(\gamma,\lambda,\mu)\frac{1}{\sqrt{2}(-z)^{\frac{1}{2}}}+C_0(\gamma,\lambda,\mu)+O((-z)^{\frac{1}{2}}),\ as\ z\to 2+,$$

where $\mathrm{C}(\gamma,\lambda,\mu)=-\gamma-\lambda-\mu+\gamma\lambda+\lambda\mu+2\gamma\mu-\gamma\lambda\mu.$

Proof. The above asymptotics for the determinant $\Delta_{\gamma\lambda\mu}(z)$ can be derived by simply calculations and taking into account the asymptotics of the functions $a(\cdot),b(\cdot),c(\cdot),d(\cdot),e(\cdot)$ and $f(\cdot)$ in Lemma 3.

Statement of the main results and their proofs.

Let we introduce the following set:

$$\mathbb{G}_{3,+}=\{(\gamma,\lambda,\mu)\in\mathbb{R}^3\colon \mathrm{C}(\gamma,\lambda,\mu)<0,\ A(\gamma,\lambda)>0,\gamma>1\},$$

where $\mathrm{C}(\gamma,\lambda,\mu)=-\gamma-\lambda-\mu+\gamma\lambda+\lambda\mu+2\gamma\mu-\gamma\lambda\mu$ and $\mathrm{A}(\gamma,\lambda)=1-\lambda-2\gamma+\gamma\lambda=0.$

Theorem . Let $\gamma,\lambda,\mu\in\mathbb{R}$. If $(\gamma,\lambda,\mu)\in\mathbb{G}_{3,+}$the operator $H_{\gamma\lambda\mu}$ has three eigenvalues above the essential spectrum and has no eigenvalues below the essential spectrum.

Proof. Let we assume $(\gamma,\lambda,\mu)\in\mathbb{G}_{3,+}$. It means $\mathrm{C}(\gamma,\lambda,\mu)<0,\ A(\gamma,\lambda)>0,\gamma>1$. According to positivity of γ and Remark 3. the function $\Delta_{\gamma 00}(z)=1+\gamma a(z)$ is continuous and monotone increasing in $(2,+\infty)$. Moreover $\mathrm{C}(\gamma,0,0)=-\gamma<0.$

Lemma 4 yields that $\lim\limits_{z\to 2+}\Delta_{\gamma 00}(z)=-\infty,\ \lim\limits_{z\to+\infty}\Delta_{\gamma 00}(z)=1.$

Hence the function $\Delta_{\gamma 00}(z)$ has exactly one zero above the essential spectrum. Let we denote this zero by z_{11}. Since to monotonicity of the function we can find followings:
$1+\gamma a(z)<0,\ \ if\ \ 2<z<z_{11}$ and $1+\gamma a(z)>0,\ \ if\ \ z>z_{11}.$

Next we prove that the function $\Delta_{\gamma\lambda 0}(z)$ has two zeros greater than 2. To prove this fact we show that $\Delta_{\gamma\lambda 0}(z_{11})<0$ as follow:

$$\Delta_{\gamma\lambda 0}(z_{11})=\big(1+\gamma a(z_{11})\big)\big(1+\lambda d(z_{11})\big)-\gamma\lambda b^2(z_{11})=-\gamma\lambda b^2(z_{11})<0.$$

Due to $(\gamma,\lambda,\mu)\in\mathbb{G}_{3,+}$ we can find that $-\gamma-\lambda+\gamma\lambda=\mathrm{A}(\gamma,\lambda)+(\gamma-1)>0.$

Equality $\mathrm{C}(\gamma,\lambda,0)=-\gamma-\lambda+\gamma\lambda$ and Lemma 4. yields that

$\lim\limits_{z\to 2+}\Delta_{\gamma\lambda 0}(z)=+\infty,\ \Delta_{\gamma\lambda 0}(z_{11})<0,\ \lim\limits_{z\to+\infty}\Delta_{\gamma\lambda 0}(z)=1.$

Hence the function $\Delta_{\gamma\lambda 0}(z)$ has two zeros above the essential spectrum. Let we denote these zeros by z_{21} and z_{22}.

In the below we prove that the function $\Delta_{\gamma\lambda\mu}(z)$ has three zeros greater than 3. Due to definition

$$\Delta_{\gamma\lambda\mu}(z)=\ \Delta_{\gamma\lambda 0}(z)\big(1+\mu f(z)\big)+2\gamma\lambda\mu b(z)c(z)e(z)-\gamma\mu(1+\lambda d(z))c^2(z)-\lambda\mu(1+\gamma a(z))e^2(z).$$

It is easy to check that

$$\Delta_{\gamma\lambda 0}(z_{21})=(1+\gamma a(z_{21}))(1+\lambda d(z_{21}))-\gamma\lambda(b(z_{21}))^2=0.$$

Therefore

$$\Delta_{\gamma\lambda\mu}(z_{21})=\mu(\sqrt{\gamma}\sqrt{-(1+\lambda d(z_{21}))}c(z_{21})+\sqrt{\lambda}\sqrt{-(1+\gamma a(z_{21}))}e(z_{21}))^2>0.$$

Similarly can be shown

$$\Delta_{\gamma\lambda\mu}(z_{21})=-\mu(\sqrt{\gamma}\sqrt{1+\lambda d(z_{21})}c(z_{21})+\sqrt{\lambda}\sqrt{1+\gamma a(z_{21})}e(z_{21}))^2<0.$$

Due to above inequalities and Lemma 4 we can find following relations

$$\lim_{z\to 2+} \Delta_{\gamma\lambda\mu}(z) = -\infty,\ \Delta_{\gamma\lambda\mu}(z_{21}) > 0,\ \Delta_{\gamma\lambda\mu}(z_{22}) < 0, \qquad \lim_{z\to+\infty} \Delta_{\gamma\lambda\mu}(z) = 1.$$

Then the above relations yield the existence of zeros z_{31}, z_{32} and z_{33} of the function $\Delta_{\gamma\lambda\mu}(z)$ satisfying the following inequalities:

$$2 < z_{31} < z_{21} < z_{32} < z_{22} < z_{33}.$$

Hence, if $(\gamma, \lambda, \mu) \in \mathbb{G}_3{}^+$ then the function $\Delta_{\gamma\lambda\mu}(z)$ has three roots greater than 2. The Lemma 1 gives that the operator $H_{\gamma\lambda\mu}$ has three eigenvalues above the essential spectrum. Moreover, since to the perturbation operator $V_{\gamma\lambda\mu}$, $\gamma, \lambda, \mu \in \mathbb{R}$ is of rank at most 3, the operator $H_{\gamma\lambda\mu}$ has at most three eigenvalues outside the essential spectrum. Thus, the operator $H_{\gamma\lambda\mu}$ has no one eigenvalues below the essential spectrum.

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