ISNN:2181-1296



SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ ШАРОФА РАШИДОВА

SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

ILMIY AXBOROTNOMA

НАУЧНЫЙ ВЕСТНИК SCIENTIFIC JOURNAL

ANIQ VA TABIIY FANLAR SERIYASI

Matematika, Mexanika, Informatika
Fizika, Kimyo, Biologiya, Geografiya

№ 3/2 (139)
2023

ISSN 2181-1296

ILMIY AXBOROTNOMA

НАУЧНЫЙ ВЕСТНИК SCIENTIFIC JOURNAL

2023-yil, 3-son (2-seriya)(139) ANIQ VA TABIY FANLAR SERIYASI

Matematika, Mexanika, Informatika, Fizika, Kimyo, Biologiya, Geografiya

Samarqand viloyat matbuot boshqarmasida ro‘yxatdan o‘tish tartibi 09-25.
Jurnal 1999-yildan chop qilina boshlagan va OAK ro‘yxatiga kiritilgan.

BOSH MUHARRIR **R. I. XALMURADOV, t.f.d., professor**
BOSH MUHARRIR O‘RINBOSARLARI: **H.A. XUSHVAQTOV, f-m.f.d., dotsent**
A. M. NASIMOV, t.f.d., professor

TAHRIRIYAT KENGASHI:

ANIQ FANLAR

SH.A.ALIMOV - O‘zFA akademigi
S.N.LAKAYEV - O‘zFA akademigi (SamDU)
M.M.MIRSAIDOV - O‘zFA akademigi
A.S.SOLEEV - f.-m.f.d., professor (SamDU)
I.A.IKROMOV - f.-m.f.d., professor (SamDU)
B.X.XO‘JAYAROV - f.-m.f.d., professor (SamDU)
A.G.YAGOLA - f.-m.f.d., professor (Moskva davlat universiteti, Rossiya)
I.I.JUMANOV - f.-m.f.d., professor (SamDU)
X.X.XUDOYNAZAROV - t.f.d., professor (SamDU)
ALBERTO DEL BIMBO - Florensiya universiteti professori, Italiya
L.SABIROV - f.-m.f.d., professor (SamDU)
A.JUMABOYEV - f.-m.f.d., professor (SamDU)
N.N.NIZAMOV - f.-m.f.d., professor (SamDU)
O.Q.QUVONDIQOV - f.-m.f.d., professor (SamDU)
I.A.RAXMATULLAYEV - f.-m.f.d., professor
A.SH.YARMUXAMEDOV - f.-m.f.n., (SamDU)
X.S.XAYDAROV - f.-m.f.n., dotsent (SamDU)

TABIY FANLAR

M.X.ASHUROV - O‘zFA akademigi
N.B. FERAPONTOV - k.f .d., professor (Moskva davlat universiteti, Rossiya)
SH. M. TUGIZOV - professor, Koliforniya universiteti, AQSh
H. I. AKBAROV - k.f.d., professor (O'zMU)
E. A. ABDURAXMONOV - k.f.d., professor (SamDU)
N. K. MUXAMADIYEV - k.f.d., professor (SamDU)
L. A. BULAVIN - Kiev milliy universiteti professori, Ukraina
X. Q. XAYDAROV - b.f.d., professor (SamDU)
Z. I. IZZATULLAYEV - b.f.d., professor (SamDU)
Sh. T. XOLIQULOV - g.f.d., professor (SamDU)
S. B. ABBASOV - g.f.d., professor (SamDU)
Q. S. YARASHEV - g.f.d., professor (SamDU)
GUN-SIK PARK - Seul univeriteti professori, Koreya
D.B.XURSANOV - g.f.f.d., dotsent (SamDU)
M. S. QUZIYEV - b.f.f.d., dotsent (SamDU)

Obuna indeksi – yakka tartibdagi obunachilar uchun **- 5583,**
tashkilot, korxonalar uchun **- 5584**

MUNDARIJA	
Кулжанов У.Н., Адилов А.А. *Спектралные свойства одночастичного оператора шредингера с контактным потенциалом*……..	3-7
Eshbekov R. *The cauchy problem for the modified korteweg-de vries equation with finite density in the class of periodic functions*……..	8-16
Abdurakhmonov B., Zaripov O., Teshayeva M. *Lagranj ko`paytuvchilari yordamida tengsizliklarni isbotlash*……	17-21
Umirzoqova I., Boboxonova G. *Oshkormas funksiya egriligi va kvadratik formalari*……..	22-26
Qurbonov H.G‘., Rustamov M.K., Gafurova D.A., Temiron N.N., Toshpulatova M.Sh. *Synthesis and properties of difficult combustion material based on modified polyacrylonitrile*……	27-32
Холиков Д.Ш., Худайбердиев З.Б., Исмоилов Х.Ф. *Упрощение нелинейного уравнения крутильных колебаний круглого конического слоя*……	33-38
Kamoliddinov D.U. *Density and regular distribution*……	39-43
Kuliev K., Eshimova M., Turaqulov T. *A new method for norm of a hardy-volterra operator*……	44-50
Kuziev B.N. *A comparative study on feature selection methods in QSAR*……..	51-55
Мардонов Ж.А. *Задача коши для неоднородного лапласова поля*……	56-61
Нуритдинов И., У.Эсанов З., Саидахмедов К.Х., Мажидов Х.М. *Преобразовании Dy1 центров в Dy2 центры в кристаллах* Dy_2SiO_5……	62-66
Akhmadova M.O. *The number of eigenvalues of the discrete schrödinger operator*……	67-71
Курбаниязов А.А., Сырлыбекқызы С., Раманкулов Ш. Ж., Аймбетова И.О., Ещанова Т.Д. *Новые данные высокого временного разрешения о морских и их изменчивости в зависимости от климатических условий*……	72-77
Ikromov I.A., Safarov A.R., Khudoyberdiev D. *Invariant estimates for generalized oscillatory integrals with polynomial phase*……	78-86
Urazbayev A.K. *Amudaryo hozirgi deltasidagi yer usti suv oqimlarining geografik strukturalari*……	87-91
Komilova N.K., Karshibayeva L.K. *Iqlim o‘zgarishlari va aholi kasalliklarining hududiy tarkibi (Jizzax va Sirdaryo viloyatlari misolida)*……	92-99
Нуритдинов И., Эсанов З.У., Саидахмедов К.Х., Ирисов Ш. *Влияние ионизирующего излучения на спектрально-люминесцентные свойства органических сцинтилляторов ups-923a*……	100-104
Alimkulov N.R. *Antropogen landshaftlarni tadqiq etish asosida landshaft-ekologik sharoit tushunchasining shakllanishi*……	105-113
Кувандиков О.К., Усаров У.Т. *Электрические, гальваномагнитные свойства редкого металла при различных температурах*	114-125
Komilova N.K., Ravshanov A.X. *Global iqlim o‘zgarishlari sharoitida O‘zbekistonda yuqumli-parazitar kasalliklar tarqalishining hududiy jihatlari*……	126-132

Рахматуллаев И.А., Ботиров Х.З., Чернега Н.В., Хайдаров Х.С. *Двухфотонно-возбуждаемая люминесценция в полициклических ароматических соединениях.....*	133-138
Anarova Sh.A., Jabbarov J.S., Rustamov A.A. Kattalashtirish masshtabiga bog'liq bo'lmagan fraktal o'lchovlarni geometrik modellashtirish…….	138-142
Nurmuhamedova D.K., Kutlieva G.J., Sohibnazarova X.A., Zairova X.T. *Mahalliy sut achituvchi bakteriyalarning antimikrob va bakteriosinogen faolligini tadqiq etish …….*	143-148
Тайланов Н.А., Уринов С.Х. *Численное моделирование уравнения гросса-питаевского.....*	149-159

УДК 517.511 **DOI: https://doi.org/10.59251/2181-1296.2023.v3.139.2.2149**

LAGRANJ KO`PAYTUVCHILARI YORDAMIDA TENGSIZLIKLARNI ISBOTLASH

B.Abdurakhmonov[1], O.Zaripov[1], M.Teshayeva[2]

[1]*Sharof Rashidov nomidagi Samarqand davlat universiteti*
[2]*O`zbekiston-Finlandiya pedagogika instituti*
E-mail: abduraxmanovbobomurod258@gmail.com

Annotatsiya. Xalqaro Matematika Olimpiadalarida tengsizliklarni isbotlash alohida o`rin tutadi. Bu kabi tengsizliklarni isbotlashning turli usullari mavjud bo`lib, ushbu maqolada ishlatilishi sodda va qulay hisoblangan Lagranj ko`paytuvchilari yordamida tengsizliklarni isbotlash keltirilgan.

**Kalit so`zlar:** Lagranj ko`paytuvchilari, shartli ekstremum, ekstremumning yetarli sharti.

Proving inequalities using the Lagrange multipliers

Abstract. Proving inequalities has an important role in the International Mathematical Olympiad. There are various methods of proving such inequalities, and this paper presents the proof of inequalities using Lagrange multipliers, which are considered simple and convenient to use.

Keywords: Lagrange multipliers, conditional extremum, sufficient condition of extremum.

Доказательство неравенств с помощью множителей Лагранжа

Аннотация. Доказательство неравенств играет важную роль в Международной математической олимпиаде. Существуют различные методы доказательства таких неравенств, и в данной работе представлено доказательство неравенств с использованием множителей Лагранжа, которые считаются простыми и удобными в использовании.

Ключевые слова: множители Лагранжа, условный экстремум, достаточное условие экстремума.

Ma’lumki, bir va ko’p o’zgaruvchili funksiyalarning ekstremumlarini topish masalasi amaliyotda dolzarb hisoblanadi. Shuningdek, ko’p o’zgaruvchili funksiyaning ma’lum bir shartlar asosida topilgan ekstremumi optimallashtirish masalalarini yechishda, amaliyotda foydalanishda o’z ahamiyatiga egadir[3]. Xususan, ushbu turdagi funksiyalarning shartli ekstremumlarini topishni matematik olimpiada masalalarida uchraydigan tengizliklarni isbotlashda ham qo`llash mumkin[4]. Quyida shartli ekstremum, Lagranj ko`paytuvchilari haqida tushuncha, shuningdek, ushbu ko`paytuvchilar yordamida Xalqaro Matematik Olimpiada tengsizliklarini isbotlash usuli keltirilgan.

Ta’rif. Faraz qilaylik, n o’zgaruvchili $f(x)$ funksiya a nuqtaning biror atrofida aniqlangan bo’lsin. Agar a nuqtaning shunday atrofi topilib, bu atrofdan olingan istalgan x argument qiymatlari uchun $f(a) \geq f(x)\ \big(f(a) \leq f(x)\big)$ tengsizlik bajarilsa, u holda f funksiya $a \in \mathbb{R}^n$ nuqtada *lokal maksimum (lokal minimum) ga* ega deyiladi.

Endilikda biz berilgan funksiyaning ekstremal qiymatlarini biror qo’shimcha shartlar bajarilganda topish masalasini o’rganamiz. Bunda asosan qo’shimcha shartlar o’zgaruvchilarning qiymatlarini cheklash shaklida beriladi[1].

Masalan $n-$ o’zgaruvchili $f(x_1, x_2, \dots, x_n)$ funksiya maksimal yoki minimal qiymatini funksiya argumentlari

$$\varphi_1(x_1, x_2, \dots, x_n) = 0, \varphi_2(x_1, x_2, \dots, x_n) = 0, \dots, \varphi_k(x_1, x_2, \dots, x_n) = 0\ (n > k)$$

qo’shimcha shartlarni qanoatlantirganda topish masalasini qaraylik. Bunda $\varphi_i(x_1, x_2, \dots, x_n), (\ i = \overline{1, k})$ funksiya ikki marta differensiallanuvchi funksiyadir. Bu holda topilgan maksimum qiymat *shartli maksimum*, minimum qiymat esa *shartli minimum* deyiladi. Shartli maksimum va shartli minimum birgalikda *shartli ekstremum* deb ataladi.

Shartli ekstremumlarni topish uchun asosan *Lagranj ko’paytuvchilar* usulidan foydalaniladi. Ya’ni shunday $\exists \mu_1, \mu_2, \dots, \mu_k$ sonlar tanlab olinib, quyidagi Lagranj funksiyasi tuziladi:

$$L(x_1, x_2, \dots, x_n, \mu_1, \mu_2, \dots, \mu_k) = f(x_1, x_2, \dots, x_n) - \sum_{i=1}^{k} \mu_i \varphi_i(x_1, x_2, \dots, x_n)$$

So’ngra, Lagranj funksiyasining barcha xususiy hosilalarini nolga tenglashtirishdan hosil bo’lgan quyidagi tenglamalar sistemasi yechiladi[1]:

$$\begin{cases} \dfrac{\partial L(x_1, x_2, \dots, x_n, \mu_1, \mu_2, \dots, \mu_k)}{\partial x_1} = 0 \\ \dfrac{\partial L(x_1, x_2, \dots, x_n, \mu_1, \mu_2, \dots, \mu_k)}{\partial x_2} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{\partial L(x_1, x_2, \dots, x_n, \mu_1, \mu_2, \dots, \mu_k)}{\partial x_n} = 0 \\ \varphi_1(x_1, x_2, \dots, x_n) = 0 \\ \varphi_2(x_1, x_2, \dots, x_n) = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \varphi_k(x_1, x_2, \dots, x_n) = 0 \end{cases}$$

Ushbu $(n+k)$ ta noma`lumli tenglamalar sistemasini yechish orqali topilgan $A(x_1, x_2, \dots, x_n)$ nuqta $f(x_1, x_2, \dots, x_n)$ funksiyaning shartli ekstremum nuqtasi bo'ladi. Topilgan nuqtaning shartli maksimum yoki shartli minimum ekanligini aniqlash uchun shartli ekstremumning yetarli shartlaridan foydalaniladi.

Shartli ekstremumning yetarli shartlarini quyidagicha keltirish mumkin[2]:
$A(x_1, x_2, \dots, x_n)$ nuqta $f(x_1, x_2, \dots, x_n)$ funksiyaning shartli ekstremum nuqtasi bo'lsin. $\varphi_i(x_1, x_2, \dots, x_n), i = \overline{1,k}, k < n$ funksiyalar A nuqtaning biror atrofida uzluksiz ikkinchi tartibli xususiy hosilalarga ega va quyidagi F matritsaning rangi k ga teng bo'lsin:

$$F = \begin{pmatrix} \dfrac{\partial \varphi_1(A)}{\partial x_1} & \dfrac{\partial \varphi_1(A)}{\partial x_2} & \dots & \dfrac{\partial \varphi_1(A)}{\partial x_n} \\ \dfrac{\partial \varphi_2(A)}{\partial x_1} & \dfrac{\partial \varphi_2(A)}{\partial x_2} & \dots & \dfrac{\partial \varphi_2(A)}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \dfrac{\partial \varphi_k(A)}{\partial x_1} & \dfrac{\partial \varphi_k(A)}{\partial x_2} & \dots & \dfrac{\partial \varphi_k(A)}{\partial x_n} \end{pmatrix}.$$

Agar $d^2L(A)$ kvadratik forma $musbat\,(manfiy)$ aniqlangan bo'lsa, A nuqta $f(x_1, x_2, \dots, x_n)$ funksiyaning $\varphi_i(x_1, x_2, \dots, x_n) = 0, i = \overline{1,k}$ cheklov tenglamalarini qanoatlantirgandagi $shartli\ minimum\ (shartli\ maksimum)$ nuqtasi bo'ladi.

Algebra kursidan ma'lumki, ushbu

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_i x_j, \qquad (a_{ij} = a_{ji}), \qquad a_{ij} \in R\,.$$

kvadratik formaning musbat aniqlangan bo'lishi uchun kvadratik forma matritsasining bosh minorlari musbatligi, manfiy aniqlangan bo`lishi uchun esa bosh minorlarining ishoralari almashinib(manfiy ishoradan boshlab) kelishi yetarli.

Biz ko'nikma hosil qilishimiz uchun dastlab $f(x,y)$ funksiya $\varphi(x,y) = 0$ shart bilan berilganda shartli ekstremumga tekshirish masalasini ko'rib chiqamiz.
Yuqoridagi yetarlilik shartiga ko'ra, $d^2L(A)$ kvadratik formaning musbat yoki manfiy aniqlanganlikka tekshiramiz. Malumki, $d^2L(A) = f_x''(A)dx^2 + 2f_{xy}''(A)dxdy + f_y''(A)dy^2$ tenglik o'rinli. $\varphi(x,y) = 0$ tenglikdan $\frac{dx}{dy} = -\frac{\varphi_y'(A)}{\varphi_x'(A)}$ kelib chiqishini e'tiborga olsak, kvadratik formaning ko'rinishi quyidagicha bo'ladi:

$$d^2L(A) = dy^2\left(f''(A)\frac{dx^2}{dy^2} + 2f_{xy}''(A)\frac{dx}{dy} + f_y''(A)\right) =$$

$$= \frac{dy^2}{(\varphi_x')^2}\left(f_x''(A)\left(\varphi_y'(A)\right)^2 - 2f_{xy}''(A)\varphi_x'(A)\varphi_y'(A) + f_y''(A)\left(\varphi_x'(A)\right)^2\right) =$$

$$= -\frac{dy^2}{(\varphi_x'(A))^2}\begin{vmatrix} 0 & \varphi_x'(A) & \varphi_y'(A) \\ \varphi_x'(A) & L_{xx}''(A) & L_{xy}''(A) \\ \varphi_y'(A) & L_{yx}''(A) & L_{yy}''(A) \end{vmatrix}.$$

Quyidagicha belgilash kiritadigan bo'lsak:

$$H(A)=\begin{vmatrix} 0 & \varphi_x'(A) & \varphi_y'(A) \\ \varphi_x'(A) & L_{xx}''(A) & L_{xy}''(A) \\ \varphi_y'(A) & L_{yx}''(A) & L_{yy}''(A) \end{vmatrix},$$

$d^2L(A)$ kvadratik formaning ishorasi $H(A)$ determinantning ishorasiga bog'liq bo'ladi ya'ni, agar $H(A)>0$ bo'lsa, $f(x,y)$ funksiya shartli maksimumga, aks holda shartli minimumga erishadi.

Endi $f(x_1,x_2,\dots,x_n)$ funksiyani $\varphi_i(x_1,x_2,\dots,x_n)=0,\ i=\overline{1,k}, k<n$ shartlar bilan berilganda shartli ekstremumga tekshirish masalasini qaraylik. Bu holda ham $d^2L(A)$ kvadratik formaning ishorasi quyidagi H determinantga bog'liq bo'ladi:

$$H=\begin{vmatrix} 0 & 0 & \dots & 0 & \frac{\partial\varphi_1}{\partial x_1} & \frac{\partial\varphi_1}{\partial x_2} & \dots & \frac{\partial\varphi_1}{\partial x_n} \\ 0 & 0 & \dots & 0 & \frac{\partial\varphi_2}{\partial x_1} & \frac{\partial\varphi_2}{\partial x_2} & \dots & \frac{\partial\varphi_2}{\partial x_n} \\ \cdot & \cdot & \dots & \cdot & \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & 0 & \frac{\partial\varphi_k}{\partial x_1} & \frac{\partial\varphi_k}{\partial x_2} & \dots & \frac{\partial\varphi_k}{\partial x_n} \\ \frac{\partial\varphi_1}{\partial x_1} & \frac{\partial\varphi_2}{\partial x_1} & \dots & \frac{\partial\varphi_k}{\partial x_1} & \frac{\partial^2L}{\partial^2x_1} & \frac{\partial^2L}{\partial x_1\partial x_2} & \dots & \frac{\partial^2L}{\partial x_1\partial x_n} \\ \frac{\partial\varphi_1}{\partial x_2} & \frac{\partial\varphi_2}{\partial x_2} & \dots & \frac{\partial\varphi_k}{\partial x_2} & \frac{\partial^2L}{\partial x_2\partial x_1} & \frac{\partial^2L}{\partial^2x_2} & \dots & \frac{\partial^2L}{\partial x_2\partial x_n} \\ \cdot & \cdot & \dots & \cdot & \cdot & \cdot & \dots & \cdot \\ \frac{\partial\varphi_1}{\partial x_n} & \frac{\partial\varphi_2}{\partial x_n} & \dots & \frac{\partial\varphi_k}{\partial x_n} & \frac{\partial^2L}{\partial x_n\partial x_1} & \frac{\partial^2L}{\partial x_n\partial x_2} & \dots & \frac{\partial^2L}{\partial^2x_n} \end{vmatrix},$$

Agar H matritsaning $H_{k+1},H_{k+2},\dots,H_{k+n}$ minorlarning A nuqtadagi qiymatlari ishorasi $(-1)^k$ ning ishoralari bilan bir xil bo'lsa, u holda A nuqta shartli minimum nuqta bo'ladi.

Agar H matritsaning $H_{k+1},H_{k+2},\dots,H_{k+n}$ minorlarning A nuqtadagi qiymatlari ishorasi navbat bilan o'zgarsa va $H_{k+1}(A)$ ning ishorasi $(-1)^{k+1}$ bilan bir xil bo'lsa, u holda A nuqta shartli maksimum nuqta bo'ladi. Ushbu determinantni shartli ravishda "yetarlilik determinant" deb ataymiz.

Quyida Lagranj ko`paytuvchilari yordamida yechish mumkin bo'lgan tengsizliklar keltirilgan.

1-misol. Quyidagi tengsizlikni isbotlang:

$$\frac{x^n+y^n}{2}\geq\left(\frac{x+y}{2}\right)^n$$

bu yerda $n\geq 1$ va $x\geq 0, y\geq 0$.

Isbot. Dastlab quyidagicha belgilash kiritaylik: $x+y=a,\ a\in R^+$.

Masala shartiga ko'ra, $f(x,y)=\frac{x^n+y^n}{2}$ funksiyaning argumentlari

$$\varphi(x,y)=x+y-a=0,$$

shartni qanoatlantirganda shartli minimum qiymatini topishimiz yetarli.

Lagranj funksiyasini tuzamiz:

$$L(x,y,\mu)=\frac{x^n+y^n}{2}-\mu(x+y-a),$$

bu yerda $x,y\in\mathbb{R}^+$.

Tuzilgan Lagranj funksiyasining xususiy hosilalaridan foydalanib, quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases}\frac{\partial L(x,y,\mu)}{\partial x}=\frac{nx^{n-1}}{2}-\mu=0\\ \frac{\partial L(x,y,\mu)}{\partial y}=\frac{ny^{n-1}}{2}-\mu=0\\ x+y-a=0\end{cases}$$

Bu tenglamalar sistemasidan $x=y=\frac{a}{2}$ tenglikni aniqlaymiz. Topilgan $A\left(\frac{a}{2},\frac{a}{2}\right)$ nuqtani ekstremumning yetarlilik shartiga tekshiramiz. Buning uchun yuqorida ta`rifi keltirilgan "yetarlilik determinanti"ning $A\left(\frac{a}{2},\frac{a}{2}\right)$ nuqtadagi ishorasini aniqlaymiz. Buning uchun dastlab quyidagi hisoblashlarni bajaramiz:

1) $\varphi'_x = 1,\ \ \varphi'_y = 1;$

2) $L''_{xx}|_A = \dfrac{n(n-1)x^{n-2}}{2}\Big|_A = \dfrac{n(n-1)a^{n-2}}{2^{n-1}} > 0, \quad L''_{xy} = L''_{yx} = 0,$

$L''_{yy}|_A = \dfrac{n(n-1)y^{n-2}}{2}\Big|_A = \dfrac{n(n-1)a^{n-2}}{2^{n-1}} > 0.$

Agar $\frac{n(n-1)a^{n-2}}{2^{n-1}} = b > 0$ almashtirish olsak, u holda “yetarlilik determinant” quyidagi ko`rinishda bo`ladi:

$$H|_A = \begin{vmatrix} 0 & 1 & 1 \\ 1 & b & 0 \\ 1 & 0 & b \end{vmatrix} = -2b < 0.$$

Demak, A nuqtada $f(x,y)$ funksiya shartli minimumga erishadi.
Bundan esa berilgan tengsizlik kelib chiqadi:

$$f(x,y) = \frac{x^n + y^n}{2} \geq \frac{1}{2}\left(\left(\frac{a}{2}\right)^n + \left(\frac{a}{2}\right)^n\right) = \left(\frac{a}{2}\right)^n = \left(\frac{x+y}{2}\right)^n.$$

2-misol[Gabriel Dospinescu, Marian Tetiva]. Agar $x, y, z \in R^+, x + y + z = xyz$ bo’lsa, quyidagi tengsizlikni isbotlang:

$$(x-1)(y-1)(z-1) \leq 6\sqrt{3} - 10.$$

Isbot. Masala shartiga ko’ra, $f(x,y,z) = (x-1)(y-1)(z-1)$ funksiyaning argumentlari $\varphi(x,y,z) = xyz - x - y - z = 0$ shartni qanoatlantirganda shartli maksimum qiymatini topish yetarli.
Dastlab, Lagranj funksiyasini tuzib olamiz:

$$L(x,y,z) = (x-1)(y-1)(z-1) + \mu(xyz - x - y - z).$$

Lagranj funksiyasining xususiy hosilalaridan foydalanib, quyidagi tenglamalar sistemasini yechamiz:

$$\begin{cases} \dfrac{\partial L(x,y,z,\mu)}{\partial x} = (y-1)(z-1) + \mu(yz-1) = 0 \\ \dfrac{\partial L(x,y,z,\mu)}{\partial y} = (x-1)(z-1) + \mu(xz-1) = 0 \\ \dfrac{\partial L(x,y,z,\mu)}{\partial z} = (x-1)(y-1) + \mu(xy-1) = 0 \\ xyz - x - y - z = 0 \end{cases}$$

Bu tenglamalar sistemasini yechib, quyidagilarni topamiz:

$$x = y = z = \sqrt{3}, \quad \mu = -\frac{(\sqrt{3}-1)^2}{2}.$$

Topilgan $A(\sqrt{3},\sqrt{3},\sqrt{3})$ nuqtani ekstremumning yetarlilik shartiga tekshiramiz. Buning uchun shartli ekstremumning yetarlilik shartiga tekshiramiz. Dastlab, quyidagi hisoblashlarni bajaramiz:

1) $\varphi'_x = (yz-1)|_A = 2,\ \ \varphi'_y = (xz-1)|_A = 2,\ \ \varphi'_z = (xy-1)|_A = 2;$

2) $F''_{xx} = F''_{yy} = F''_{zz} = 0,\ \ F''_{xy} = F''_{yx} = (z - 1 + \mu z)|_A = \frac{(\sqrt{3}-1)^2}{2},$

$F''_{xz} = F''_{zx} = (y - 1 + \mu y)|_A = \frac{(\sqrt{3}-1)^2}{2},\ \ F''_{yz} = F''_{zy} = (x - 1 + \mu x)|_A = \frac{(\sqrt{3}-1)^2}{2}.$

Agar $\frac{(\sqrt{3}-1)^2}{2} = a$ almashtirish olsak:

$$H_2(A) = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4 < 0,\ \ H_3(A) = \begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & a \\ 2 & a & 0 \end{vmatrix} = 8a > 0$$

$$H_3(A) = \begin{vmatrix} 0 & \varphi'_x & \varphi'_y & \varphi'_z \\ \varphi'_x & F''_{xx} & F''_{xy} & F''_{xz} \\ \varphi'_y & F''_{yx} & F''_{yy} & F''_{yz} \\ \varphi'_z & F''_{zx} & F''_{zy} & F''_{zz} \end{vmatrix}_A = \begin{vmatrix} 0 & 2 & 2 & 2 \\ 2 & 0 & a & a \\ 2 & a & 0 & a \\ 2 & a & a & 0 \end{vmatrix} = -12a^2 < 0.$$

Demak, $f(x,y,z)$ funksiya A nuqtada shartli maksimumga erishadi. Bundan esa berilgan tengsizlikni hosil qilamiz: $(x-1)(y-1)(z-1) \leq 6\sqrt{3} - 10.$

3-misol. Agar $x, y -$ natural sonlar bo’lsa, quyidagi tengsizlikni isbotlang:

$$\sqrt[x+y]{x^y y^x} \leq \frac{x+y}{2}.$$

Isbot. Dastlab quyidagicha belgilash kiritamiz: $x + y = a \in \mathbb{N}.$

Masala shartiga ko'ra, $f(x,y)=\sqrt[a]{x^y y^x}$ funksiyaning argumentlari $\varphi(x,y)=x+y-a=0$ shartni qanoatlantirganda shartli maksimum qiymatini topishimiz kerak. Lagranj funksiyasini tuzib olamiz:

$$L(x,y,\mu)=\sqrt[a]{x^y y^x}-\mu(x+y-a).$$

Lagranj funksiyasining xususiy hosilalaridan foydalanib, quyidagi tenglamalar sistemasini tuzamiz:

$$\begin{cases}\dfrac{\partial L(x,y,\mu)}{\partial x}=\dfrac{y}{a}y^{\frac{x}{a}-1}x^{\frac{y}{a}}+\dfrac{lny}{a}x^{\frac{y}{a}}y^{\frac{x}{a}}-\mu=0\\ \dfrac{\partial L(x,y,\mu)}{\partial y}=\dfrac{x}{a}x^{\frac{y}{a}-1}y^{\frac{x}{a}}+\dfrac{lnx}{a}y^{\frac{x}{a}}x^{\frac{y}{a}}-\mu=0\\ x+y-a=0\end{cases}$$

Bu tenglamalar sistemasining yechimi $x=y=\frac{a}{2}$ bo`lishini aniqlash qiyin emas.

Topilgan $A\left(\frac{a}{2},\frac{a}{2}\right)$ nuqtani ekstremumning yetarlilik shartiga tekshiramiz. Buning uchun quyidagi "yetarlilik determinant"ning $A\left(\frac{a}{2},\frac{a}{2}\right)$ nuqtadagi ishorasini aniqlaymiz. Quyidagi hisoblashlarni bajaramiz:

1) $\varphi'_x=1,\quad \varphi'_y=1;$

2) $L''_{xx}|_A=\left(\frac{y}{a}\left(\frac{y}{a}-1\right)x^{\frac{y}{a}-2}y^{\frac{x}{a}}+\frac{2ylny}{a^2}x^{\frac{y}{a}-1}y^{\frac{x}{a}}+\frac{ln^2y}{a^2}x^{\frac{y}{a}}y^{\frac{x}{a}}\right)\Big|_A=-\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a},$

$$L''_{xy}|_A=L''_{yx}|_A=\left(\frac{1}{a}y^{\frac{x}{a}-1}x^{\frac{y}{a}}+\frac{y}{a}\left(\left(\frac{x}{a}-1\right)y^{\frac{x}{a}-2}x^{\frac{y}{a}}+\frac{lnx}{a}y^{\frac{x}{a}-1}x^{\frac{y}{a}}\right)+\frac{lnxlny}{a^2}x^{\frac{y}{a}}y^{\frac{x}{a}}++\frac{xlny}{a^2}x^{\frac{y}{a}}y^{\frac{x}{a}-1}\right)|_A$$
$$=\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a},$$

$$L''_{yy}|_A=\left(\frac{x}{a}\left(\frac{x}{a}-1\right)y^{\frac{x}{a}-2}x^{\frac{y}{a}}+\frac{2xlnx}{a^2}y^{\frac{x}{a}-1}x^{\frac{y}{a}}+\frac{ln^2x}{a^2}y^{\frac{x}{a}}x^{\frac{y}{a}}\right)\Big|_A=-\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a}.$$

Agar $-\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a}=b$ va $\frac{1}{2a}+\frac{lna-ln2}{a}+\frac{(lna-ln2)^2}{2a}=c$ almashtirishlarni olib, $c-b=\frac{1}{a}$ tenglikni e`tiborga olsa, u holda "yetarlilik detarminanti" quyidagi ko`rinishda bo`ladi:

$$H|_A=\begin{vmatrix}0 & \varphi'_x & \varphi'_y\\ \varphi'_x & L''_{xx} & L''_{xy}\\ \varphi'_y & L''_{yx} & L''_{yy}\end{vmatrix}_A=\begin{vmatrix}0 & 1 & 1\\ 1 & b & c\\ 1 & c & b\end{vmatrix}=2(c-b)=\frac{2}{a}>0.$$

Demak, $f(x,y,z)$ funksiya A nuqtada shartli maksimumga erishadi. Bundan esa quyidagi munosabat kelib chiqadi:

$$f(x,y)=\sqrt[x+y]{x^y y^x}\le\sqrt[a]{\left(\frac{a}{2}\right)^{\frac{a}{2}}\left(\frac{a}{2}\right)^{\frac{a}{2}}}=\frac{a}{2}=\frac{x+y}{2}.$$

Maktab o`quvchilari o`rtasida o`tkaziladigan turli Matematika Olimpiadalarida, xususan, Xalqaro Matematika Olimpiada(IMO) masalalarida tengsizliklarni isbotlashda yuqoridagi kabi Lagranj ko`paytuvchilaridan foydalanish qulay va bir muncha soddadir. Ko`rinib turibdiki, ushbu tushuncha elementar matematika masalalarini Oliy matematika yordamida hal qilish imkonini beradi.

Adabiyotlar

1. Alimov Sh., Ashurov R. Matematik tahlil. 2-qism. "Mumtoz so`z", Toshkent, 2018.
2. Азларов Т., Мансуров Ҳ. Математик анализ 2-қисм. "Ўқитувчи", Тошкент 1989.
3. Тер-Крикоров А.М. , Шабунин.М.И. Курс математического анализа: Учеб.пособие для вузов. – 3 -е изд.,исправл. –М.:ФИЗМАТ-ЛИТ, 2001.
4. Демидович Б.П. Сборник задач и математическому анализу. "Наука", Москва 1972.
5. Eshbekov R.Kh., Abdurakhmonov B.G. Solve mathematical Olympiad problems using the Rolle's theorem. Poland Science Journal, 2021. Volume 12. №2, 4-9 pp.
6. https://math.libretexts.org/Bookshelves/Calculus/Book%3A_Calculus_(OpenStax)/14%3A_Differentiation_of_Functions_of_Several_Variables.

SAMARQAND UNIVERSITETI ILMIY AXBOROTNOMASI

НАУЧНЫЙ ВЕСТНИК **SCIENTIFIC REPORTS**

Texnik muharrir **Sh.Xoliqulov**

Muharrirlar:

I.Sulaymonov - fil.f.d., dotsent
E.Arziqulov - f.-m.f.d., dotsent
O.Yusupova - fil.f.n., dotsent

Mas'ul muharrirlar:

A. Sh. Yarmuxamedov - f.-m.f.n.
X. S. Haydarov - f.-m.f.n., dotsent

Muassis: Samarqand davlat universiteti Manzil: 140104,
Samarqand shahri, Universitet xiyoboni, 15.
Telefon: (0 366) 240-38-47, Faks: (0 366) 239-13-87 e-mail:
axborotnoma.uz
SamDU «Ilmiy axborotnoma» jurnali tahririyati kompyuterida terildi. Bosishga 30.06.2023-yilda ruxsat etildi. Qog'oz o'lchami A-4. Nashriyot hisob tabog'i 10,00.
Buyurtma raqami 570. Adadi 30 nusxa.

ISSN 2181-1296

