ISNN:2181-1296



SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

САМАРКАНДСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ ИМЕНИ ШАРОФА РАШИДОВА

SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

ILMIY AXBOROTNOMA

НАУЧНЫЙ ВЕСТНИК SCIENTIFIC JOURNAL

ANIQ VA TABIIY FANLAR SERIYASI

Matematika, Mexanika, Informatika
Fizika, Kimyo, Biologiya, Geografiya

№ 3/2 (139)
2023

ISSN 2181-1296

ILMIY AXBOROTNOMA

НАУЧНЫЙ ВЕСТНИК SCIENTIFIC JOURNAL

2023-yil, 3-son (2-seriya)(139) ANIQ VA TABIY FANLAR SERIYASI

Matematika, Mexanika, Informatika, Fizika, Kimyo, Biologiya, Geografiya

Samarqand viloyat matbuot boshqarmasida ro‘yxatdan o‘tish tartibi 09-25.
Jurnal 1999-yildan chop qilina boshlagan va OAK ro‘yxatiga kiritilgan.

BOSH MUHARRIR **R. I. XALMURADOV, t.f.d., professor**
BOSH MUHARRIR O‘RINBOSARLARI: **H.A. XUSHVAQTOV, f-m.f.d., dotsent**
A. M. NASIMOV, t.f.d., professor

TAHRIRIYAT KENGASHI:

ANIQ FANLAR

SH.A.ALIMOV - O‘zFA akademigi
S.N.LAKAYEV - O‘zFA akademigi (SamDU)
M.M.MIRSAIDOV - O‘zFA akademigi
A.S.SOLEEV - f.-m.f.d., professor (SamDU)
I.A.IKROMOV - f.-m.f.d., professor (SamDU)
B.X.XO‘JAYAROV - f.-m.f.d., professor (SamDU)
A.G.YAGOLA - f.-m.f.d., professor (Moskva davlat universiteti, Rossiya)
I.I.JUMANOV - f.-m.f.d., professor (SamDU)
X.X.XUDOYNAZAROV - t.f.d., professor (SamDU)
ALBERTO DEL BIMBO - Florensiya universiteti professori, Italiya
L.SABIROV - f.-m.f.d., professor (SamDU)
A.JUMABOYEV - f.-m.f.d., professor (SamDU)
N.N.NIZAMOV - f.-m.f.d., professor (SamDU)
O.Q.QUVONDIQOV - f.-m.f.d., professor (SamDU)
I.A.RAXMATULLAYEV - f.-m.f.d., professor
A.SH.YARMUXAMEDOV - f.-m.f.n., (SamDU)
X.S.XAYDAROV - f.-m.f.n., dotsent (SamDU)

TABIY FANLAR

M.X.ASHUROV - O‘zFA akademigi
N.B. FERAPONTOV - k.f .d., professor (Moskva davlat universiteti, Rossiya)
SH. M. TUGIZOV - professor, Koliforniya universiteti, AQSh
H. I. AKBAROV - k.f.d., professor (O'zMU)
E. A. ABDURAXMONOV - k.f.d., professor (SamDU)
N. K. MUXAMADIYEV - k.f.d., professor (SamDU)
L. A. BULAVIN - Kiev milliy universiteti professori, Ukraina
X. Q. XAYDAROV - b.f.d., professor (SamDU)
Z. I. IZZATULLAYEV - b.f.d., professor (SamDU)
Sh. T. XOLIQULOV - g.f.d., professor (SamDU)
S. B. ABBASOV - g.f.d., professor (SamDU)
Q. S. YARASHEV - g.f.d., professor (SamDU)
GUN-SIK PARK - Seul univeriteti professori, Koreya
D.B.XURSANOV - g.f.f.d., dotsent (SamDU)
M. S. QUZIYEV - b.f.f.d., dotsent (SamDU)

Obuna indeksi – yakka tartibdagi obunachilar uchun **- 5583,**
tashkilot, korxonalar uchun **- 5584**

MUNDARIJA	
Кулжанов У.Н., Адилов А.А. *Спектралные свойства одночастичного оператора шредингера с контактным потенциалом*……..	3-7
Eshbekov R. *The cauchy problem for the modified korteweg-de vries equation with finite density in the class of periodic functions*……..	8-16
Abdurakhmonov B., Zaripov O., Teshayeva M. *Lagranj ko`paytuvchilari yordamida tengsizliklarni isbotlash*……	17-21
Umirzoqova I., Boboxonova G. *Oshkormas funksiya egriligi va kvadratik formalari*……..	22-26
Qurbonov H.G‘., Rustamov M.K., Gafurova D.A., Temiron N.N., Toshpulatova M.Sh. *Synthesis and properties of difficult combustion material based on modified polyacrylonitrile*……	27-32
Холиков Д.Ш., Худайбердиев З.Б., Исмоилов Х.Ф. *Упрощение нелинейного уравнения крутильных колебаний круглого конического слоя*……	33-38
Kamoliddinov D.U. *Density and regular distribution*……	39-43
Kuliev K., Eshimova M., Turaqulov T. *A new method for norm of a hardy-volterra operator*……	44-50
Kuziev B.N. *A comparative study on feature selection methods in QSAR*……..	51-55
Мардонов Ж.А. *Задача коши для неоднородного лапласова поля*……	56-61
Нуритдинов И., У.Эсанов З., Саидахмедов К.Х., Мажидов Х.М. *Преобразовании Dy1 центров в Dy2 центры в кристаллах* Dy_2SiO_5……	62-66
Akhmadova M.O. *The number of eigenvalues of the discrete schrödinger operator*……	67-71
Курбаниязов А.А., Сырлыбекқызы С., Раманкулов Ш. Ж., Аймбетова И.О., Ещанова Т.Д. *Новые данные высокого временного разрешения о морских и их изменчивости в зависимости от климатических условий*……	72-77
Ikromov I.A., Safarov A.R., Khudoyberdiev D. *Invariant estimates for generalized oscillatory integrals with polynomial phase*……	78-86
Urazbayev A.K. *Amudaryo hozirgi deltasidagi yer usti suv oqimlarining geografik strukturalari*……	87-91
Komilova N.K., Karshibayeva L.K. *Iqlim o‘zgarishlari va aholi kasalliklarining hududiy tarkibi (Jizzax va Sirdaryo viloyatlari misolida)*……	92-99
Нуритдинов И., Эсанов З.У., Саидахмедов К.Х., Ирисов Ш. *Влияние ионизирующего излучения на спектрально-люминесцентные свойства органических сцинтилляторов ups-923a*……	100-104
Alimkulov N.R. *Antropogen landshaftlarni tadqiq etish asosida landshaft-ekologik sharoit tushunchasining shakllanishi*……	105-113
Кувандиков О.К., Усаров У.Т. *Электрические, гальваномагнитные свойства редкого металла при различных температурах*	114-125
Komilova N.K., Ravshanov A.X. *Global iqlim o‘zgarishlari sharoitida O‘zbekistonda yuqumli-parazitar kasalliklar tarqalishining hududiy jihatlari*……	126-132

Рахматуллаев И.А., Ботиров Х.З., Чернега Н.В., Хайдаров Х.С. *Двухфотонно-возбуждаемая люминесценция в полициклических ароматических соединениях.....*	133-138
Anarova Sh.A., Jabbarov J.S., Rustamov A.A. Kattalashtirish masshtabiga bog'liq bo'lmagan fraktal o'lchovlarni geometrik modellashtirish…….	138-142
Nurmuhamedova D.K., Kutlieva G.J., Sohibnazarova X.A., Zairova X.T. *Mahalliy sut achituvchi bakteriyalarning antimikrob va bakteriosinogen faolligini tadqiq etish …….*	143-148
Тайланов Н.А., Уринов С.Х. *Численное моделирование уравнения гросса-питаевского.....*	149-159

УДК 517.957 **DOI: https://doi.org/10.59251/2181-1296.2023.v3.139.2.2148**

THE CAUCHY PROBLEM FOR THE MODIFIED KORTEWEG-DE VRIES EQUATION WITH FINITE DENSITY IN THE CLASS OF PERIODIC FUNCTIONS

R. Eshbekov
Samarkand State University named after Sharof Rashidov
E-mail: raykhonbek@samdu.uz

Abstract. In this paper, the method of inverse spectral problems is used to integrate the nonlinear modified Korteweg-de Vries equation with finite density in the class of periodic functions. The solvability of the Cauchy problem for an infinite system of Dubrovin differential equations in the class of fivefold continuously differentiable periodic functions is proven. It is shown that the sum of a uniformly convergent functional series constructed by solving the system of Dubrovin equations and the first trace formula satisfies the mKdV equations with finite density.

Keywords: modified Korteweg-de Vries equation with finite density, Dirac operator, spectral data, system of Dubrovin differential equations, trace formulas.

Davriy funksiyalar sinfida chekli zichlikka ega modifitsrlangan Korteveg-de Friz tenglamasi uchun Koshi masalasi

Annotatsiya. Ushbu maqolada teskari spektral masalalar usuli davriy funksiyalar sinfida chekli zichlikka ega modifitsrlangan Korteveg-de Friz tenglamasini integrallashga qo`llanilgan. Cheksiz sondagi Dubrovin tenglamalar sistemasi uchun qo`yilgan Koshi masalasining yechimga ega ekanligi besh marta uzluksiz differensiallanuvchi funksiyalar sindida isbotlangan. Dubrovin tenglamalar sistemasining yechimi orqali topilgan funksional qatorlarning tekis yaqinlashuvchilgi hamda birinchi izlar formulasi chekli zichlikka ega mKdF tenglamasini qanoatlantirishi ko`rsatilgan.

Kalit so`zlar: chekli zichlikka ega modifitsrlangan Korteveg-de Friz tenglamasi, Dirak operatori, spektral berilganlar, Dubrovin differensial tenglamalar sistemasi, izlar formulasi.

Задача Коши для модифицированного уравнения Кортевега-де Фриза с конечной плотности в классе периодических функций

Аннотация. В данной работе метод обратной спектральной задачи применяется для интегрирования нелинейного модифицированного уравнения Кортевега-де Фриза с конечной плотности в классе периодических функций. Доказано разрешимость задачи Коши для бесконечной системы дифференциальных уравнений Дубровина в классе пятикратно непрерывно дифференцируемых периодических функций. Показано, что сумма равномерно сходящегося функционального ряда построенного с помощью решения в системы уравнений Дубровина и формула первого следа, удовлетворяет уравнения мКдФ с конечной плотности.

Ключевые слова: модифицированное уравнения Кортевега-де Фриза с конечной плотности, оператор Дирака, спектральные данные, система дифференциальных уравнений Дубровина, формулы следов.

Inverse spectral problems play an important role in the integration of some important evolutionary equations of mathematical physics. An important breakthrough was made by Gardner, Green, Kruskal, and Miura [1] in 1967, where they discovered a deep connection between the well-known nonlinear Korteweg-de Vries (KdV) equation

$$q_t = 6qq_x - q_{xxx}, \;\; q(x,t)|_{t=0} = q_0(x), x \in \mathbb{R}, \;\; t > 0,$$

and the spectral theory of the Sturm-Liouville operator

$$\mathcal{L}(t)y \equiv -y'' + q(x,t)y = \lambda y, \;\; x \in \mathbb{R}, \;\; t > 0.$$

The authors of [1] managed to find a global solution of the Cauchy problem for the KdV equation by reducing it to an inverse problem of scattering theory. The inverse scattering theory problem for the Sturm-Liouville operator on the whole line was studied in the works of Faddeev[2], Marchenko[3], Levitan[4] and others. In [5], Lax showed the universality of the inverse scattering problem method (ISPM) and generalized the equation KdV, introducing the concept of the higher KdV equation. In their original paper [6], Zakharov and Shabat showed that the following nonlinear Schrodinger equation (NSE)

$$iu_t \pm 2|u|^2u + u_{xx} = 0$$

can also be included in the ISPM formalism. Using the technique proposed by Lax, they found the NSE solution for given initial functions $u(x,0)$ that sufficiently rapidly decreased at $|x| \to \infty$. Soon Wadati [7], based on the ideas of Zakharov and Shabat, proposed a method for solving the Cauchy problem for the modified KdV (mKdV) equation

$$u_t \pm 6u^2u_x + u_{xxx} = 0.$$

It is well known that finding an explicit formula for solving the nonlinear evolution equation of KdV and the modified equation of KdV(mKdV) in the class of periodic functions essentially depends on the number of nontrivial gaps in the spectrum of the Sturm-Liouville and periodic Dirac operators. In this regard, the class of periodic functions is conveniently divided into two sets:

1. The class of periodic finite-gap functions; 2. The class of periodic infinite-gap functions.

Using the method of the inverse spectral problem for Sturm-Liouville and Dirac operators with periodic potentials, when the spectrum has only a finite number of nontrivial gaps, in the works of Its-Matveev[8], Dubrovin-Novikov[9], Its-Kotlyarov[10], Matveev-Smirnov [11], the complete integrability of the equation (KdV, NSE, mKdV) was established in the class of finite-gap periodic and quasiperiodic functions. In addition, for finite-gap solutions of equations (KdV, NSE, mKdV, etc.), an explicit formula was derived in terms of the Riemann theta functions.

Thus, in these papers [8]–[11], the solvability of the Cauchy problem for nonlinear equations (KdV, NSE, mKdV, etc.) was proven for any finite-gap initial data.

It was shown (see [12]) that if the initial conditions are in fourfold differentiable periodic infinite-gap zone functions for the KdV equation, then the Cauchy problem has a smooth solution at all times $t > 0$. This is proven by analysing the Dubrovin equations for a dvisor.

In this paper, the solvability of problems (1) and (2) is established as periodic and infinite gap functions in the class $C^n(\mathbb{R})$, $n \geq 5$. From this, in particular $\rho = 0$, follows the result of the work [13].

Consider the Cauchy problem for the modified Korteweg-de Vries equation with finite density of the following form:

$$\begin{cases} q_t = q_{xxx} - 6(q^2 - \rho^2)q_x, \ \ 0 < \rho < \infty \\ q(x,t)|_{t=0} = q_0(x), \qquad q_0(x+\pi) = q_0(x) \in C^5(\mathbb{R}) \end{cases} \tag{1}$$

in the class of real infinite-gap $\pi-$ periodic with respect to x functions:

$$q(x+\pi,t) = q(x,t), \ \ x \in \mathbb{R}, \ \ t > 0,$$
$$p(x,t), q(x,t) \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0). \tag{2}$$

In this paper, we propose an algorithm for constructing the exact solution $q(x,t), x \in \mathbb{R}, t > 0$ of problem (1)-(2) by reducing it to an inverse spectral problem for the following Dirac operator:

$$\mathcal{L}(\tau,t)y \equiv B\frac{dy}{dx} + \Omega(x+\tau,t)y = \lambda y, \ \ x \in \mathbb{R}, \tau \in \mathbb{R}, t > 0, \tag{3}$$

where

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \ \Omega(x+\tau,t) = \begin{pmatrix} 0 & q(x+\tau,t) \\ q(x+\tau,t) & 0 \end{pmatrix}, \ \ y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}.$$

Denote by $c(x,\lambda,\tau,t) = \big(c_1(x,\lambda,\tau,t), c_2(x,\lambda,\tau,t)\big)^T$ and $s(x,\lambda,\tau,t) = \big(s_1(x,\lambda,\tau,t), s_2(x,\lambda,\tau,t)\big)^T$ solutions of equation (3) with initial conditions $c(0,\lambda,\tau,t) = (1,0)^T$ and $s(0,\lambda,\tau,t) = (0,1)^T$. The function $\Delta(\lambda,\tau,t) = c_1(\pi,\lambda,\tau,t) + s_2(\pi,\lambda,\tau,t)$ is called the Lyapunov function for equation (3).

The spectrum of the Dirac operator $\mathcal{L}(\tau,t)$ is purely continuous and consists of the following set:

$$\sigma(\mathcal{L}) = \mathbb{R}\backslash\left(\bigcup_{n=-\infty}^{+\infty}(\lambda_{2n-1},\lambda_{2n})\right).$$

The intervals $(\lambda_{2n-1},\lambda_{2n}), n \in \mathbb{Z}$ are called gaps, where λ_n are the roots of the equation $\Delta(\lambda) \mp 2 = 0$.

They coincide with the eigenvalues of the periodic or antiperiodic $\big(y(0) = \pm y(\pi)\big)$ problem for equation (4). We denote the roots of equation $s_1(\pi,\lambda,\tau,t) = 0$ by $\xi_n(\tau,t), n \in \mathbb{Z}$, while $\xi_n(\tau,t) \in [\lambda_{2n-1},\lambda_{2n}], n \in \mathbb{Z}$. The numbers $\xi_n(\tau,t), n \in \mathbb{Z}$ and the signs $\sigma_n(\tau,t) = sign\{s_2(\pi,\xi_n,\tau,t) - c_1(\pi,\xi_n,\tau,t)\}$, $n \in \mathbb{Z}$ are called spectral parameters of the operator $\mathcal{L}(\tau,t)$. The spectral parameters

$\xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}$ and the spectrum boundaries $\lambda_n(\tau,t), n \in \mathbb{Z}$ are called the spectral data of the Dirac operator $\mathcal{L}(\tau,t)$.

The problem of recovering the coefficient $\Omega(x,t)$ of the operator $\mathcal{L}(\tau,t)$ from spectral data is called the inverse problem. The coefficient $q(x,t)$ of the operator $\mathcal{L}(\tau,t)$ is determined uniquely from the spectral data $\{\lambda_n(\tau,t), \xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}\}$. If using the initial function $q_0(x+\tau), \tau \in \mathbb{R}$, we construct the Dirac operator $\mathcal{L}(\tau,0)$ of the following form:

$$\mathcal{L}(\tau,0)y \equiv B\frac{dy}{dx} + \Omega_0(x+\tau)y = \lambda y, \quad x \in \mathbb{R}, \quad \tau \in \mathbb{R},$$

$$\Omega_0(x+\tau) = \begin{pmatrix} 0 & q_0(x+\tau) \\ q_0(x+\tau) & 0 \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},$$

then we will see that the boundaries of the spectrum $\lambda_n(\tau), n \in \mathbb{Z}$ do not depend on the parameter $\tau \in \mathbb{R}$, i.e., $\lambda_n(\tau) = \lambda_n, n \in \mathbb{Z}$, and the spectral parameters depend on the parameter τ: $\xi_n^0 = \xi_n^0(\tau), \sigma_n^0 = \sigma_n^0(\tau) = \pm 1$, which are also periodic functions: $\xi_n^0(\tau+\pi) = \xi_n^0(\tau)$, $\sigma_n^0(\tau+\pi) = \sigma_n^0(\tau), \tau \in \mathbb{R}, n \in \mathbb{Z}$. Solving the direct problem, we find the spectral data $\{\lambda_n, \xi_n^0(\tau), \mathrm{y}_n^0(\tau) = \pm 1, n \in \mathbb{Z}\}$ of the operator $\mathcal{L}(\tau,0)$.

The main result of this paper is contained in the following theorem:

Theorem 1. Let $q(x,t), x \in \mathbb{R}, t > 0$ is the solution of the Cauchy problem (1)-(2). Then, the boundaries of the spectrum $\lambda_n(\tau,t), n \in \mathbb{Z}$ of the operator $\mathcal{L}(\tau,t)$ do not depend on τ and t, i.e., $\lambda_n(\tau,t) = \lambda_n, n \in \mathbb{Z}$ and the spectral parameters$\xi_n(\tau,t), \sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}$, satisfy the first and second equations of the system of Dubrovin differential equations, respectively:

$$1.\ \frac{\partial \xi_n(\tau,t)}{\partial \tau} = 2(-1)^{n-1}\sigma_n(\tau,t)h_n\big(\xi(\tau,t)\big)\xi_n(\tau,t), n \in \mathbb{Z}; \tag{4}$$

$$2.\ \frac{\partial \xi_n(\tau,t)}{\partial t} = 2(-1)^{n}\sigma_n(\tau,t)h_n\big(\xi(\tau,t)\big)g_n\big(\xi(\tau,t)\big), n \in \mathbb{Z}. \tag{5}$$

Here, the signs $\sigma_n(\tau,t) = \pm 1, n \in \mathbb{Z}$ changes to the opposite at each collision of the point $\xi_n(\tau,t), n \in \mathbb{Z}$ with the boundaries of its gap $[\lambda_{2n-1}, \lambda_{2n}]$. In addition, the following initial conditions are satisfied

$$\xi_n(\tau,t)|_{t=0} = \xi_n^0(\tau), \quad \sigma_n(\tau,t)|_{t=0} = \sigma_n^0(\tau), n \in \mathbb{Z} \tag{6}$$

where $\xi_n^0(\tau), \sigma_n^0(\tau) = \pm 1, n \in \mathbb{Z}$ are spectral parameters of the Dirac operator $\mathcal{L}(\tau,0)$. Sequences $h_n(\xi)$ are $g_n(\xi), n \in \mathbb{Z}$ are determined by the following formulas:

$$h_n(\xi) = \sqrt{(\xi_n(\tau,t) - \lambda_{2n-1})\big(\lambda_{2n} - \xi_n(\tau,t)\big)} \cdot f_n(\xi),$$

$$f_n(\xi) = \sqrt{\prod_{k=-\infty, k\neq n}^{+\infty} \frac{\big(\lambda_{2k-1} - \xi_n(\tau,t)\big)\big(\lambda_{2k} - \xi_n(\tau,t)\big)}{\big(\xi_k(\tau,t) - \xi_n(\tau,t)\big)^2}}, \tag{7}$$

$$g_n(\xi) = 4\xi_n^3(\tau,t) + 2\xi_n(\tau,t)(q^2(\tau,t) + q_\tau(\tau,t) - 3\rho^2), \quad n \in \mathbb{Z}. \tag{8}$$

Proof. Let $q(x,t), x \in \mathbb{R}, t > 0$ is π periodic function with respect to x, satisfy equation (1). Denote by $y_n = \big(y_{n,1}(x,\tau,t), y_{n,2}(x,\tau,t)\big)^T, n \in \mathbb{Z}$ the orthonormal eigenfunctions of the operator $\mathcal{L}(\tau,t)$ considered on the interval $[0;\pi]$ with the Dirichlet boundary conditions

$$y_1(0,\tau,t) = 0, \quad y_1(\pi,\tau,t) = 0,$$

corresponding to the eigenvalues $\xi_n = \xi_n(\tau,t), n \in \mathbb{Z}$. Differentiating with respect to t the following identity:

$$\xi_n(\tau,t) = (L(\tau,t)y_n, y_n), \quad n \in \mathbb{Z},$$

and using the symmetry of the operator $\mathcal{L}(\tau,t)$,, we have

$$\frac{\partial \xi_n(\tau,t)}{\partial t} = \big(\dot{\Omega}(x+\tau,t)y_n, y_n\big), \quad n \in \mathbb{Z}. \tag{9}$$

Using the following explicit form of the scalar product

$$(y,z) = \int_0^\pi \big[y_1(x)\overline{z_1(x)} + y_2(x)\overline{z_2(x)}\big]dx, \quad y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}, \quad z = \begin{pmatrix} z_1(x) \\ z_2(x) \end{pmatrix},$$

equality (9) can be written in the following form:

$$\frac{\partial \xi_n(\tau,t)}{\partial t} = \int_0^\pi 2y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)q_t(x+\tau,t)dx. \qquad (10)$$

Putting expression (1) into (10), we obtain the following equality:

$$\frac{\partial \xi_n(\tau,t)}{\partial t} = I_1(\tau,t) + I_2(\tau,t) + I_3(\tau,t)$$

where

$$I_1(\tau,t) = 2\int_0^\pi y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)q_{xxx}(x+\tau,t)dx,$$

$$I_2(\tau,t) = -12\int_0^\pi y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)q^2(x+\tau,t)q_x(x+\tau,t)dx,$$

$$I_3(\tau,t) = 12\rho^2\int_0^\pi y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)q_x(x+\tau,t)dx.$$

Using the following identities

$$\begin{cases} y'_{n,1} = qy_{n,1} - \xi_n y_{n,2} \\ y'_{n,2} = \xi_n y_{n,1} - qy_{n,2} \end{cases},$$

It is easy to check that the following equalities are valid:

$$\begin{cases} \left(y_{n,1}\cdot y_{n,2}\right)' = \xi_n\left(y_{n,1}^2 - y_{n,2}^2\right) \\ \left(y_{n,1}^2 - y_{n,2}^2\right)' = 2q\left(y_{n,1}^2 + y_{n,2}^2\right) - 4\xi_n y_{n,1}y_{n,2} \\ \left(y_{n,1}^2 + y_{n,2}^2\right)' = 2q\left(y_{n,1}^2 - y_{n,2}^2\right) \end{cases}$$

where

$$y_{n,1} = y_{n,1}(x,\tau,t),\ \ y_{n,2} = y_{n,2}(x,\tau,t),\ \ q = q(x+\tau,t),\ \ \xi_n = \xi_n(\tau,t).$$

Using these equalities and boundary conditions $y_{n,1}(0,\tau,t) = y_{n,1}(\pi,\tau,t) = 0$, it is easy to calculate $I_1(\tau,t)$:

$$I_1(\tau,t) = 2\int_0^\pi y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)q_{xxx}(x+\tau,t)dx = 2\int_0^\pi y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)dq_{xx}(x+\tau,t) =$$

$$= -2\xi_n(\tau,t)\int_0^\pi \left(y_{n,1}^2(x,\tau,t) - y_{n,2}^2(x,\tau,t)\right)dq_x = 2\xi_n(\tau,t)q_\tau(\tau,t)\left[y_{n,2}^2(\pi,\tau,t) - y_{n,2}^2(0,\tau,t)\right] +$$

$$+2\xi_n(\tau,t)\int_0^\pi \left(y_{n,1}^2(x,\tau,t) + y_{n,2}^2(x,\tau,t)\right)dq^2(x+\tau,t)$$

$$- 8\xi_n^2(\tau,t)\int_0^\pi y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)dq(x+\tau,t) =$$

$$= 2\xi_n(\tau,t)\{q_\tau(\tau,t) + q^2(\tau,t)\}\left[y_{n,2}^2(\pi,\tau,t) - y_{n,2}^2(0,\tau,t)\right] -$$

$$-4\xi_n(\tau,t)\int_0^\pi q^3(x+\tau,t)\left\{y_{n,1}^2(x,\tau,t) - y_{n,2}^2(x,\tau,t)\right\}dx +$$

$$+4\xi_n^3(\tau,t)\int_0^\pi 2q(x+\tau,t)\left\{y_{n,1}^2(x,\tau,t) - y_{n,2}^2(x,\tau,t)\right\}dx =$$

$$= 2\xi_n(\tau,t)\{4\xi_n^3(\tau,t) + q_\tau(\tau,t) + q^2(\tau,t)\}\left[y_{n,2}^2(\pi,\tau,t) - y_{n,2}^2(0,\tau,t)\right] -$$

$$-4\xi_n(\tau,t)\int_0^\pi q^3(x+\tau,t)\left\{y_{n,1}^2(x,\tau,t) - y_{n,2}^2(x,\tau,t)\right\}dx.$$

Then, calculate $I_2(\tau,t)$ and $I_3(\tau,t)$:

$$I_2(\tau,t)=-12\int_0^{\pi} y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)q^2(x+\tau,t)q_x(x+\tau,t)dx=$$

$$=-4\int_0^{\pi} y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)dq^3(x+\tau,t)$$

$$=4\xi_n(\tau,t)\int_0^{\pi} q^3(x+\tau,t)\{y_{n,1}^2(x,\tau,t)-y_{n,2}^2(x,\tau,t)\}dx.$$

$$I_3(\tau,t)=12\rho^2\int_0^{\pi} y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)q_x(x+\tau,t)dx=12\rho^2\int_0^{\pi} y_{n,1}(x,\tau,t)y_{n,2}(x,\tau,t)dq(x+\tau,t)$$

$$=$$

$$=-6\rho^2\xi_n(\tau,t)\int_0^{\pi} 2q(x+\tau,t)\{y_{n,1}^2(x,\tau,t)-y_{n,2}^2(x,\tau,t)\}dx$$

$$=-6\rho^2\xi_n(\tau,t)\left[y_{n,2}^2(\pi,\tau,t)-y_{n,2}^2(0,\tau,t)\right].$$

Thus,

$$\frac{\partial\xi_n(\tau,t)}{\partial t}=2\xi_n(\tau,t)\{4\xi_n^3(\tau,t)+q_\tau(\tau,t)+q^2(\tau,t)-6\rho^2\xi_n(\tau,t)\}\left[y_{n,2}^2(\pi,\tau,t)-y_{n,2}^2(0,\tau,t)\right]. \quad (11)$$

Since the eigenvalues $\xi_n(\tau,t)$ of the Dirichlet problem for equation (3) are simple, then the following equality holds:

$$y_n(x,\tau,t)=\frac{1}{c_n(\tau,t)}s(x,\xi_n(t),\tau,t),$$

where $c_n^2(\tau,t)=\int_0^{\pi}[s_1^2(x,\xi_n(t),\tau,t)+s_2^2(x,\xi_n(t),\tau,t)]dx=-\frac{\partial s_1(\pi,\xi_n(t),\tau,t)}{\partial\lambda}\cdot s_2(\pi,\xi_n(t),\tau,t)$.

Using these equalities, we have

$$y_{n,2}^2(\pi,\tau,t)-y_{n,2}^2(0,\tau,t)=-\frac{s_2(\pi,\xi_n(t),\tau,t)-\frac{1}{s_2(\pi,\xi_n(t),\tau,t)}}{\frac{\partial s_1(\pi,\xi_n(t),\tau,t)}{\partial\lambda}}.$$

Putting into this relation the following equality

$$s_2(\pi,\xi_n,\tau,t)-\frac{1}{s_2(\pi,\xi_n,\tau,t)}=\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4},$$

we obtain

$$y_{n,2}^2(\pi,\tau,t)-y_{n,2}^2(0,\tau,t)=-\frac{\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n)-4}}{\frac{\partial s_1(\pi,\xi_n,\tau,t)}{\partial\lambda}}. \quad (12)$$

Given the decomposition

$$\Delta^2(\lambda)-4=-4\pi^2\prod_{k=-\infty}^{\infty}\frac{(\lambda-\lambda_{2k-1})(\lambda-\lambda_{2k})}{\alpha_k^2},\quad s_1(\pi,\lambda,t)=\pi\prod_{k=-\infty}^{\infty}\frac{\xi_k-\lambda}{\alpha_k}$$

where $\alpha_0=1$ and $\alpha_k=k$ at $k\neq 0$, rewrite equation (12) in the following form:

$$y_{n,2}^2(\pi,\tau,t)-y_{n,2}^2(0,\tau,t)=2(-1)^n\sigma_n(\tau,t)h_n(\xi).$$

Putting this expression into identity (11), we obtain (5). Similarly, we can prove (4).

If we replace the Dirichlet boundary conditions with periodic $\big(y(0,t)=y(\pi,t)\big)$ or antiperiodic $\big(y(0,t)=-y(\pi,t)\big)$ boundary conditions, then instead of equation (11), we have

$$\frac{\partial\lambda_n(\tau,t)}{\partial t}=0,\ \ i.e.\ \ \lambda_n(\tau,t)=\lambda_n(\tau,0),\ \ n\in\mathbb{Z}.$$

Now, in equation $\mathcal{L}(\tau,t)v_n=\lambda_n(\tau,t)v_n,\ n\in\mathbb{Z}\backslash\{0\}$,, we obtain $t=0$. Since the eigenvalues $\lambda_n(\tau)=\lambda_n(\tau,0),\ n\in\mathbb{Z}\backslash\{0\}$ of the periodic or antiperiodic problem for equation $\mathcal{L}(\tau,0)v_n=\lambda_n(\tau)v_n,\ n\in\mathbb{Z}$ do not depend on the parameter $\tau\in\mathbb{R}$, we have $\lambda_n(\tau,t)=\lambda_n(\tau)=\lambda_n,\ n\in\mathbb{Z}$.

The theorem has been proven.

Now, using the following trace formulas

$$q(\tau,t)=\sum_{k=-\infty}^{+\infty}(-1)^{k-1}\sigma_k(\tau,t)h_k\big(\xi(\tau,t)\big), \tag{13}$$

$$q^2(\tau,t)+q_\tau(\tau,t)=\sum_{k=-\infty}^{+\infty}\left(\frac{\lambda_{2k-1}^2+\lambda_{2k}^2}{2}-\xi_k^2(\tau,t)\right),$$

System (5) can be rewritten in the closed form:

$$\frac{\partial\xi_n(\tau,t)}{\partial t}=2(-1)^n\sigma_n(\tau,t)h_n\big(\xi(\tau,t)\big)\Bigg[4\xi_n^3(\tau,t)$$

$$+2\xi_n(\tau,t)\left(\sum_{k=-\infty}^{+\infty}\left(\frac{\lambda_{2k-1}^2+\lambda_{2k}^2}{2}-\xi_k^2(\tau,t)\right)-3\rho^2\right)\Bigg].$$

Next, using the change in variables

$$\xi_n(\tau,t)=\lambda_{2n-1}+(\lambda_{2n}-\lambda_{2n-1})\sin^2 x_n(\tau,t),\ \ n\in\mathbb{Z} \tag{14}$$

system of Dubrovin equations (5) can be rewritten as a single equation in the Banach space $\mathbb{K}$:

$$\frac{dx(\tau,t)}{dt}=H\big(x(\tau,t)\big),\ \ x(\tau,t)|_{t=0}=x^0(\tau)\in\mathbb{K}$$

where

$$\mathbb{K}=\left\{x=(\dots,x_{-1}(\tau,t),x_0(\tau,t),x_1(\tau,t),\dots):\|x\|=\sum_{n=-\infty}^{+\infty}(1+|n|)(\lambda_{2n}-\lambda_{2n-1})|x_n|<\infty\right\},$$

$$H(x)=(\dots,H_{-1}(x),H_1(x),\dots),$$

$$H_n(x)=(-1)^n\sigma_n(0)f_n(\dots,\lambda_1+(\lambda_2-\lambda_1)\sin^2 x_1(\tau,t),\dots)\cdot g_n(\dots,\lambda_1+(\lambda_2-\lambda_1)\sin^2 x_1(\tau,t),\dots)=$$
$$=(-1)^n\sigma_n(0)f_n\big(x(\tau,t)\big)g_n\big(x(\tau,t)\big).$$

From the conditions $q_0(x+\pi)=q_0(x)\in C^5(\mathbb{R})$ and the asymptotics of the eigenvalues of the periodic and antiperiodic problems for the system of Dirac equations, we obtain the following equality:

$$\lambda_{2k-1},\lambda_{2k}=k+\sum_{j=1}^{6}c_jk^{-j}\pm 2^{-5}|k|^{-5}\big|q_{2k}^5\big|+|k|^{-6}\varepsilon_k^{\pm},\ \ \gamma_k\equiv\lambda_{2k}-\lambda_{2k-1}$$

$$=\frac{\big|q_{2k}^5\big|}{2^4|k|^5}+\frac{\delta_k}{|k|^6}, \tag{15}$$

$$\sum_{k=-\infty}^{\infty}\big|q_{2k}^5\big|^2<\infty,\ \sum_{k=-\infty}^{\infty}\big(\varepsilon_k^{\pm}\big)^2<\infty,\ \ \delta_k=\varepsilon_k^{+}-\varepsilon_k^{-}.$$

Hence, taking into account $\xi_n(\tau,t)\in[\lambda_{2n-1},\lambda_{2n}]$, we have

$$\inf_{k\neq n}|\xi_n(\tau,t)-\xi_k(\tau,t)|\geq a>0.$$

Now, using inequality in the above and formula (15), we estimate the following functions:

$$|f_n(x)|,\ \ \left|\frac{\partial f_n(x)}{\partial x_m}\right|,\ \ |g_n(x)|,\ \ \left|\frac{\partial g_n(x)}{\partial x_m}\right|.$$

Lemma 1. The following estimates are valid:

$$c_1\leq|f_n(x)|\leq c_2,\ \ \left|\frac{\partial f_n(x)}{\partial x_m}\right|\leq c_3\gamma_m, \tag{16}$$

$$|g_n(x)|\leq c_4|n|^3,\ \ \left|\frac{\partial g_n(x)}{\partial x_m}\right|\leq c_5\gamma_m|n|(|m|+1) \tag{17}$$

where $c_j>0, j=1,2,3,4,5$, does not depend on the parameters m and n.

Proof. Inequality (16) was proven in [13]. Therefore, let us prove inequality (17). Using asymptotics (15) and inequalities (14), we obtain the following estimates:

$$|g_n(x)|\leq 4|\lambda_{2n-1}+\gamma_n\sin^2 x_n(\tau,t)|^3+2|\lambda_{2n-1}+\gamma_n\sin^2 x_n(\tau,t)|\times$$
$$\times\left|\sum_{k=-\infty}^{\infty}\left\{\frac{\lambda_{2k}^2-\lambda_{2k-1}^2}{2}+\gamma_k\sin^2 x_k(\tau,t)\,(2\lambda_{2k-1}-\gamma_k\sin^2 x_k)\right\}-3\rho^2\right|\leq c_4|n|^3.$$

$$\left|\frac{\partial g_n(x)}{\partial x_m}\right| = 2|\lambda_{2n-1} + \gamma_n \sin^2 x_n(\tau,t)| \cdot |2\gamma_m\lambda_{2m-1}\sin 2x_m - 4\gamma_m^2 \sin^3 x_m \cos x_m|$$
$$\leq c_5\gamma_m|n|(|m|+1).$$

Lemma 1 is proved.

Lemma 2. If the initial functions $q_0(x)$ satisfy the following condition:

$$q_0(x+\pi) = q_0(x) \in C^5(\mathbb{R}),$$

then the vector function $H(x)$ satisfies the Lipschitz condition in the Banach space $\mathbb{K}$, that is, there exists a constant $L = const > 0$ such that the following inequality holds for arbitrary elements $x, y \in \mathbb{K}$:

$$||H(x) - H(y)|| \leq L||x-y||,$$

where

$$L = A\sum_{n=-\infty}^{+\infty} \gamma_n|n|^3(1+|n|) < \infty, \qquad (18)$$

Proof. First, using Lemma 1, we estimate the derivative with respect to $x_m(\tau,t)$ of the function$F_n(x) = f_n(x)g_n(x)$:

$$\left|\frac{\partial F_n(x)}{\partial x_m}\right| \leq \left|\frac{\partial f_n(x)}{\partial x_m}\right| |g_n(x)| + |f_n(x)|\left|\frac{\partial g_n(x)}{\partial x_m}\right| \leq c_3c_4\,\gamma_m|n|^3 + c_2c_5\gamma_m|n|(|m|+1) \leq$$
$$\leq c_3c_4\,\gamma_m|n|^3(|m|+1) + c_2c_5\gamma_m|n|(|m|+1) \leq \gamma_m|n|^3(|m|+1)\left\{c_3c_4 + \frac{c_2c_5}{|n|^2}\right\} \leq$$
$$\leq \gamma_m|n|^3(|m|+1)\{c_3c_4+1\} = A\gamma_m|n|^3(|m|+1).$$

where $A = c_3c_4 + 1$ does not depend on the parameters m and n.

Next, using the following expression

$$H_n\big(x(\tau,t)\big) = (-1)^n\sigma_n^0(\tau)F_n\big(x(\tau,t)\big), \quad n \in \mathbb{Z}.$$

we obtain the following equality:

$$\big|H_n\big(x(\tau,t)\big) - H_n\big(y(\tau,t)\big)\big| = \big|F_n\big(x(\tau,t)\big) - F_n\big(y(\tau,t)\big)\big|.$$

Now, let us apply the Lagrange mean value theorem to the following function:

$$\varphi(t) = F_n\big(x + t(y-x)\big),$$

on segment $t \in [0;1]$. Then, we obtain the following equalities:

$$\varphi(1) - \varphi(0) = \varphi'(t^*),$$
$$F_n(x) - F_n(y) = \sum_{k=-\infty}^{\infty} \frac{\partial F_n(\theta)}{\partial x_k}(x_k - y_k),$$

where $\theta = x + t^*(y-x)$. Hence, it follows that

$$\big|H_n\big(x(\tau,t)\big) - H_n\big(y(\tau,t)\big)\big| = \big|F_n\big(x(\tau,t)\big) - F_n\big(y(\tau,t)\big)\big| \leq \sum_{k=-\infty}^{\infty}\left|\frac{\partial F_n(\theta)}{\partial x_k}\right| |x_k(\tau,t) - y_k(\tau,t)| \leq$$
$$\leq A|n|^3\sum_{k=-\infty}^{\infty}\gamma_k(|k|+1)|x_k(\tau,t) - y_k(\tau,t)| \leq A|n|^3\|x-y\|.$$

Now, let us estimate the following norm: $\big\|H\big(x(\tau,t)\big) - H\big(y(\tau,t)\big)\big\|$:

$$\|H(x) - H(y)\| = \sum_{n=-\infty}^{\infty}\gamma_n(|n|+1)|H_n(x) - H_n(y)| \leq \sum_{n=-\infty}^{\infty}\gamma_n(|n|+1)A|n|^3\|x-y\| =$$
$$= \|x-y\|\sum_{n=-\infty}^{\infty} A\gamma_n|n|^3(|n|+1) \leq L\|x-y\|.$$

where

$$L = A\sum_{n=-\infty}^{\infty}\gamma_n|n|^3(|n|+1) < \infty.$$

Thus, the Lipschitz condition is satisfied. Therefore, the solution of the Cauchy problem (6), (7) for all $t > 0$ and $\tau \in \mathbb{R}$ exists and is unique.

Lemma 2 is proved.

Remark 1. Theorem 1 and Lemma 2 give a method for constructing a solution to problems (1)-(2).

To do this, first find the spectral data $\lambda_n, \xi_n^0(\tau), \sigma_n^0(\tau) = \pm 1, n \in \mathbb{Z}$, of the Dirac operator $\mathcal{L}(\tau, 0)$. Denote the spectral data of operator $\mathcal{L}(\tau, t)$ by $\lambda_n, \xi_n(\tau, t), \sigma_n(\tau, t) = \pm 1, n \in \mathbb{Z}$. Then, solving Cauchy problems (5) and (6) for an arbitrary value of τ, we find $\xi_n(\tau, t), \sigma_n(\tau, t), n \in \mathbb{Z}$. After that, from the trace formulas (15), we find the function $p(\tau, t)$, i.e., solving the problem (1)-(2).

Thus, we have proved the following theorem:

Theorem 2. If the initial functions $q_0(x)$ satisfy the following condition:

$$q_0(x + \pi) = q_0(x) \in C^5(\mathbb{R}),$$

then there exists a uniquely determined solution $q(\tau, t)$ of problem (1)-(2) belonging to the class $C_x^3(t > 0) \cap C_t^1(t > 0) \cap C(t \geq 0)$,, which is defined as the sum of series (13).

Remark 2. The function $q_\tau(\tau, t)$ constructed using the system of Dubrovin equations (5)-(6) and the trace formula (13) satisfies equation (1).

Proof. First, we use the following second trace formula:

$$q^2(\tau, t) + q_\tau(\tau, t) = \sum_{k=-\infty}^{+\infty} \left(\frac{\lambda_{2k-1}^2 + \lambda_{2k}^2}{2} - \xi_k^2(\tau, t) \right) \tag{19}$$

Differentiating formula (19) with respect to t, we have

$$2q(\tau, t) q_t(\tau, t) + q_{\tau t}(\tau, t) = -\sum_{k=-\infty}^{+\infty} 2\xi_k(\tau, t) \frac{\partial \xi_k(\tau, t)}{\partial t}. \tag{20}$$

Furthermore, taking into account the system of Dubrovin equations (5), from (20), we obtain

$$2q(\tau, t) q_t(\tau, t) + q_{\tau t}(\tau, t) =$$

$$= 2 \sum_{k=-\infty}^{+\infty} \{2(-1)^k \sigma_k(\tau, t) h_k(\xi(\tau, t)) \xi_k(\tau, t)\} \left[4\xi_k^3(\tau, t) + 2\xi_k(\tau, t)(q^2(\tau, t) + q_\tau(\tau, t) - 3\rho^2) \right] =$$

$$= 2 \sum_{k=-\infty}^{+\infty} \frac{\partial \xi_k(\tau, t)}{\partial \tau} \left[4\xi_k^3(\tau, t) + 2\xi_k(\tau, t)(q^2(\tau, t) + q_\tau(\tau, t) - 3\rho^2) \right] =$$

$$= 2 \sum_{k=-\infty}^{+\infty} \left\{ \frac{\partial \xi_k^4(\tau, t)}{\partial \tau} + \frac{\partial \xi_k^2(\tau, t)}{\partial \tau} (q^2(\tau, t) + q_\tau(\tau, t) - 3\rho^2) \right\}. \tag{21}$$

Next, we calculate the sums of the following functional series:

$$\sum_{k=-\infty}^{+\infty} \frac{\partial \xi_k^4(\tau, t)}{\partial \tau}, \quad \sum_{k=-\infty}^{+\infty} \frac{\partial \xi_k^2(\tau, t)}{\partial \tau}.$$

To do this, we first define the following trace formula:

$$-\frac{q_{\tau\tau\tau}(\tau, t)}{2} - q(\tau, t) q_{\tau\tau}(\tau, t) + 2q^2(\tau, t) q_\tau(\tau, t) + q^4(\tau, t) = \sum_{k=-\infty}^{+\infty} \left(\frac{\lambda_{2k-1}^4 + \lambda_{2k}^4}{2} - \xi_k^4(\tau, t) \right). \tag{22}$$

Differentiate the trace formulas (19) and (22) with respect to τ:

$$-2qq_\tau - q_{\tau\tau} = \sum_{k=-\infty}^{+\infty} \frac{\partial \xi_k^2}{\partial \tau},$$

$$\frac{q_{\tau\tau\tau\tau}}{2} + q_\tau q_{\tau\tau} + q q_{\tau\tau\tau} - 4q q_\tau^2 - 2q^2 q_{\tau\tau} - 4q^3 q_\tau = \sum_{k=-\infty}^{+\infty} \frac{\partial \xi_k^4(\tau, t)}{\partial \tau},$$

Using these formulas from (21), we find

$$2qq_t + q_{\tau t} = q_{\tau\tau\tau\tau} + 2q_\tau q_{\tau\tau} + 2qq_{\tau\tau\tau} - 8qq_\tau^2 - 4q^2q_{\tau\tau} - 8q^3q_\tau - \{4qq_\tau + 2q_{\tau\tau}\}(q^2 + q_\tau - 3\rho^2),$$

$$2qq_t + q_{\tau t} = q_{\tau\tau\tau\tau} + 2qq_{\tau\tau\tau} - 12qq_\tau^2 - 6q^2q_{\tau\tau} - 12q^3q_\tau + 12\rho^2qq_\tau + 6\rho^2q_{\tau\tau}. \qquad (23)$$

Equation (23) can be reduced to the following linear equation:

$$z_\tau(\tau,t) + \frac{1}{2q}z(\tau,t) = \varphi(\tau,t), \qquad (24)$$

where$z(\tau,t) = q_t(\tau,t)$, $\varphi(\tau,t) = q_{\tau\tau\tau\tau} + 2qq_{\tau\tau\tau} - 12qq_\tau^2 - 6q^2q_{\tau\tau} - 12q^3q_\tau + 12\rho^2qq_\tau + 6\rho^2q_{\tau\tau}$.

It is easy to check that the following function

$$z(\tau,t) = q_{\tau\tau\tau} - 6q^2q_\tau + 6\rho^2q_\tau,$$

is a solution for equation (24).

From here and from the notation $z(\tau,t) = q_t(\tau,t)$, we obtain equation (1).

Remark 3. The uniform convergence of the series in formulas (13) and (18) follows from estimate (15).

4. Conclusion.

The inverse spectral problem method is used to integrate the modified Korteweg-de Vries equation with a finite density in the class of periodic infinite-gap functions. It is proven that if the initial conditions are fivefold differentiable periodic infinite-gap functions, then the Cauchy problem for the mKdV equation with finite density has a smooth solution at all times.

References

1. Gardner C., Green I., Kruskal M., Miura R. A method for solving the Korteweg-de Vries equation.\\ Phys. Rev. Letter.Volume 19, pp. 1095-1098, (1967).
2. Фаддеев Л.Д. Свойства S-матрицы одномерного уравнения Шредингера.\\ Тр.МИ АНСССР. Volume 73, pp. 314-336, (1964).
3. Марченко В. А. Операторы Штурма-Лиувилля и их приложения. Наукова думка, Киев, 1977.
4. Левитан Б. М. Обратные задачи Штурма-Лиувилля. Наука, Москва, 1984.
5. Lax P.D. Integrals of nonlinear equations of evolution and solitary waves,\\ Comm. Pure and Appl. Math. Volume 21, pp.467-490, (1968).
6. Захаров В. Е., Шабат А. Б. Точная теория двумерной самофокусировки в одномерной автомодуляции волн в нелинейных средах.\\ ЖЭТФ, Volume 1, pp. 118-134, (1971).
7. Wadati M. The exact solution of the modified Korteweg-de Vries equation\\ J. Phys. Soc. Japn. Volume 32, pp. 44-47, (1972).
8. Итс А. Р., Матвеев В. Б. Операторы Шредингера с конечнозонным спектром и N - солитонные решения уравнения Кортевега-де Фриза.\\ ТМФ,Volume 23, pp.51-68, (1975).
9. Дубровин Б.А., Новиков С.П. Периодический и условно периодический аналоги многосолитонных решений уравнения Кортевега-де Фриза.\\ ЖЭТФ, Volume 67, pp. 2131-2143, (1974).
10. Итс А.Р., Котляров В.П. Явные формулы для решений нелинейного уравнения Шредингера. \\ Докл. АНУССР.сер.А, Volume 11, pp.965-968, (1976).
11. Матвеев В.Б., Смирнов А. О. Решения типа «волнубийц» уравнений иерархии Абловица-Каупа-Ньюэлла-Сигура: единый подход.\\ ТМФ,Volume 186, pp.191-220, (2016).
12. Хасанов А.Б., Хасанов Т. Г. Задача Коши для уравнения Кортевега-де Фриза в классе периодических бесконечнозонных функций.\\ Записки науч. семин. ПОМИ, Volume 506, pp.258-279, (2021).
13. Хасанов А. Б., Алланазарова Т. Ж. О модифицированном уравнении Кортевега-де Фриза с нагруженным членом.\\ Укр.Мат.Журн. 2021, т.73, No11, с. 1541-1563.

SAMARQAND UNIVERSITETI ILMIY AXBOROTNOMASI

НАУЧНЫЙ ВЕСТНИК **SCIENTIFIC REPORTS**

Texnik muharrir **Sh.Xoliqulov**

Muharrirlar:

I.Sulaymonov - fil.f.d., dotsent
E.Arziqulov - f.-m.f.d., dotsent
O.Yusupova - fil.f.n., dotsent

Mas'ul muharrirlar:

A. Sh. Yarmuxamedov - f.-m.f.n.
X. S. Haydarov - f.-m.f.n., dotsent

Muassis: Samarqand davlat universiteti Manzil: 140104,
Samarqand shahri, Universitet xiyoboni, 15.
Telefon: (0 366) 240-38-47, Faks: (0 366) 239-13-87 e-mail:
axborotnoma.uz
SamDU «Ilmiy axborotnoma» jurnali tahririyati kompyuterida terildi. Bosishga 30.06.2023-yilda ruxsat etildi. Qog'oz o'lchami A-4. Nashriyot hisob tabog'i 10,00.
Buyurtma raqami 570. Adadi 30 nusxa.

ISSN 2181-1296

