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BIR JINSLI BO'LMAGAN POLIANALITIK TENGLAMA YECHIMINI BIRLIK DOIRA CHEGARASINING QISMIDAN DAVOM ETTIRISH

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Annotatsiya. Ushbu maqolada bir jinsli bo'lmagan polianalitik tenglama yechimini birlik doira chegarasining qismidagi qiymatlariga ko'ra, shu doiraga davom ettirish masalasi o'rganilgan. Bu masala yechimining mavjudlik kriteriyasi o'rnatilgan

Kalit so'zlar: polianalitik tenglama, Dirixle masalasi, Borel-Pompey formulasi, Soxotskiy-Plemel formulasi, davom ettirish formulalari, Fok-Kuni teoremasi

Продолжение решения неоднородного полианалитического уравнения с части границы единичного круга

Аннотация. В данной статье исследуется задача продолжения решения неоднородного полианалитического уравнения в единичний круг по его значениям на части границе. Установлены критерии существования решения этой задачи.

Ключевые слова: полианалитическая функция, задача Дирихле, формула Бореля-Помпея, формула Сохоцкого-Племеля, формулы продолжения, теорема Фока-Куни**.**

Continuation of the solution of the inhomogeneous polyanalytic equation the unit circle a in from part of the boundary

Annotation: In this article, we study the problem of continuation the solution of an inhomogeneous polyanalytic equation to unit circle by its values on a part of boundary. Criteria for the existence of a solution to this problem is established.

Key words: polyanalytic function, The Dirichlet problem, The Borel–Pompey formula, The Sochocki-Plemelj formula, continuation formulas, Fock-Cugny theorem.

Kompleks o'zgaruvchining $z = x + iy$ tekisligida $D = \{z : |z| < 1\}$ birlik doirani belgilaymiz va $t' = e^{i\theta'}$, $t'' = e^{i\theta''}$ nuqtalarni $\partial D, 0 < \theta' < \theta'' < 2\pi$ sohaning chegarasi birlik aylanadan olamiz, ∂D ning (t', t'') yoyini S bilan belgilaymiz.

Bir jinsli bo'lmagan polianalik tenglama uchun quyidagi davom ettirish masalasini qaraymiz:

$$\frac{\partial^n W(z)}{\partial \bar{z}^n} = f(z),\ z \in D,\ \partial_{\bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right), \tag{1}$$

$$W(z) = \varphi_0(z),\ \frac{\partial W(z)}{\partial \bar{z}} = \varphi_1(z), \ldots, \frac{\partial^{n-1} W(z)}{\partial \bar{z}^{n-1}} = \varphi_{n-1}(z), z \in S. \tag{2}$$

Ya'ni (1) tenglamaning yechimini regulyarlik sohasi chegarasi qismidagi qiymatlariga ko'ra davom ettirish masalasini qaraymiz S yoyning D sohaga nisbatan garmonik o'lchovini qaraymiz: [8]

$$\omega(z, \theta', \theta'') = \frac{1}{\pi}\arg\left(\frac{z - e^{i\theta''}}{z - e^{i\theta'}} e^{\frac{\theta' - \theta''}{2} i}\right).$$

Kompleks o'zgaruvchili analitik funksiyani soha chegarasining qismidagi qiymatlariga ko'ra, shu sohaga davom ettirish formulasini birinchi bo'lib Karleman maxsus ko'rinishdagi soha uchun hosil qildi [2]. G.M.Goluzin va V.I.Krilovlar Karleman g'oyasini ixtiyoriy sohalar uchun umumlashtirishdi.[3]. Karleman formulasiga ko'ra V.A.Fok va F.M.Kunilar analitik davom ettirish masalasi yechimining mavjudlik kriteriyasini o'rnatishdi [4]. Fok-Kuni teoremasi L.A.Ayzenberg, A.A.Shlapunov, Sh.Yarmuxamedov, T. Ishankulov va boshqa mualliflarning ishlarida umumlashtirilgan [5-7,10].

1-teorema. $f \in L_p(D),\ p > 2$ *bo'lsin.* $W-$*(1) tenglamaning* D *sohadagi regulyar yechimi bo'lib,* $\overline{D} = D \cup \partial D$ *yopiq sohada o'zining (n-1)-tartibgacha hosilalari bilan differensiallanuvchi bo'lsin va (2) shartlarni qanoatlantirsin. U holda quyidagi ekvivalent davom ettirish formulalari o'rinli:*

$$W(z) = \lim_{\sigma\to\infty}\left\{\frac{1}{2\pi i}\int_S \exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\left[\varphi_0(\zeta)+(\overline{z}-\overline{\zeta})\varphi_1(\zeta)+\ldots+\frac{(\overline{z}-\overline{\zeta})^{n-1}\varphi_{n-1}(\zeta)}{(n-1)!}\right]\frac{d\zeta}{\zeta-z}-\right.$$

$$\left.-\frac{1}{\pi}\iint_D \exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\frac{(\overline{z}-\overline{\zeta})^{n-1}}{(n-1)!}f(\zeta)\frac{d\xi d\eta}{\zeta-z}\right\},\ z\in D, \qquad (3)$$

$$W(z) = \frac{1}{2\pi i}\int_S \frac{\left[\varphi_0(\zeta)+(\overline{z}-\overline{\zeta})\varphi_1(\zeta)+\ldots+\frac{(\overline{z}-\overline{\zeta})^{n-1}\varphi_{n-1}(\zeta)}{(n-1)!}\right]}{\zeta-z}d\zeta-\frac{1}{\pi}\iint_D \frac{\frac{(\overline{z}-\overline{\zeta})^{n-1}}{(n-1)!}f(\zeta)d\xi d\eta}{\zeta-z}+$$

$$+\int_0^\infty\left[\frac{1}{2\pi i}\int_S exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\left[\varphi_0(\zeta)+(\overline{z}-\overline{\zeta})\varphi_1(\zeta)+\ldots+\frac{(\overline{z}-\overline{\zeta})^{n-1}\varphi_{n-1}(\zeta)}{(n-1)!}\right]\frac{\lambda(\zeta)-\lambda(z)}{\zeta-z}d\zeta+\right.$$

$$\left.-\frac{1}{\pi}\iint_D \exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\frac{(\overline{z}-\overline{\zeta})^{n-1}}{(n-1)!}f(\zeta)\frac{\lambda(\zeta)-\lambda(z)}{\zeta-z}d\xi d\eta\right]d\sigma,\ z\in D. \qquad (4)$$

bu yerda $\lambda(z)\in A(D)$ analitik funksiya bo'lib $\omega(z)=\mathrm{Re}\,\lambda(z)$, $\sigma>0$ musbat parametr.

Teorema isboti.(3) va (4) formulalarni ekvivalent ekanligini

$$\lim_{\sigma\to\infty}\psi(\sigma,z)=\int_0^\infty\frac{d\psi(\sigma,z)}{d\sigma}+\psi(0,z)$$

formuladan kelib chiqadi.

Shuning uchun (3) formulani isbotlaymiz. $G\in C^1(D)\cap C(\overline{D})$ funksiya uchun Borel-Pompey integral tasviri o'rinli[1]:

$$G(z)=\frac{1}{2\pi i}\int_{\partial D}\frac{G(\zeta)d\zeta}{\zeta-z}-\frac{1}{\pi}\iint_D\frac{\partial G(\zeta)}{\partial\overline{\zeta}}\frac{d\xi d\eta}{\zeta-z},\ z\in D.$$

Bu formulada

$$G(\zeta)=\sum_{k=0}^{n-1}\frac{\left(\overline{z}-\overline{\zeta}\right)^k}{k!}\frac{\partial^k F(\zeta)}{\partial\overline{\zeta}^k}\quad (F\in C^n(D)\cap C^{n-1}(\overline{D}))$$

funksiyani qo'yib

$$F(z)=\frac{1}{2\pi i}\int\limits_{\partial D}\sum_{k=0}^{n-1}\frac{\left(\overline{z}-\overline{\zeta}\right)^k}{k!}\frac{\partial^k F(\zeta)}{\partial\overline{\zeta}^k}\frac{d\zeta}{\zeta-z}-\frac{1}{\pi}\iint\limits_D\frac{\left(\overline{z}-\overline{\zeta}\right)^{n-1}}{(n-1)!}\frac{\partial^n F(\zeta)}{\partial\overline{\zeta}^n}\frac{d\xi d\eta}{\zeta-z},\ z\in D.\quad(5)$$

formulani hosil qilamiz.

(5) formulani $F(z)=W(z)exp\left[\sigma\lambda(z)\right]$ funksiyaga qo'llab

$$W(z)exp\left[\sigma\lambda(z)\right]=\frac{1}{2\pi i}\int\limits_{\partial D}\left[\sum_{k=0}^{n-1}\frac{\left(\overline{z}-\overline{\zeta}\right)^k}{k!}\frac{\partial^k W(\zeta)}{\partial\overline{\zeta}^k}\right]\frac{exp\left[\sigma\lambda(\zeta)\right]d\zeta}{\zeta-z}-$$

$$-\frac{1}{\pi}\iint\limits_D\frac{\left(\overline{\zeta}-\overline{z}\right)^{n-1}}{(n-1)!}\frac{\partial^n W(z)exp\left[\sigma\lambda(z)\right]}{\partial\overline{\zeta}^n}\frac{d\xi d\eta}{\zeta-z},$$

yoki

$$W(z)=\frac{1}{2\pi i}\int\limits_{S\cup(\partial D/S)}\sum_{k=0}^{n-1}\frac{\left(\overline{z}-\overline{\zeta}\right)^k}{k!}\frac{\partial^k W(\zeta)}{\partial\overline{\zeta}^k}\frac{\exp\left[\sigma\left\{\lambda(\zeta)-\lambda(z)\right\}\right]d\zeta}{\zeta-z}-\quad(6)$$

$$-\frac{1}{\pi}\iint\limits_D\exp\left[\sigma\left\{\lambda(\zeta)-\lambda(z)\right\}\right]f(\zeta)\frac{d\xi d\eta}{\zeta-z}\Bigg\},$$

ni olamiz. (6) dan $z\in D$ uchun

$$\Bigg|W(z)-\frac{1}{2\pi i}\int\limits_S\sum_{k=0}^{n-1}\frac{\left(\overline{z}-\overline{\zeta}\right)^k}{k!}\varphi_k(\zeta)\frac{\exp\left[\sigma\left\{\lambda(\zeta)-\lambda(z)\right\}\right]d\zeta}{\zeta-z}+$$

$$+\frac{1}{\pi}\iint\limits_D\exp\left[\sigma\left\{\lambda(\zeta)-\lambda(z)\right\}\right]\frac{\left(\overline{z}-\overline{\zeta}\right)^{n-1}}{(n-1)!}f(\zeta)\frac{d\xi d\eta}{\zeta-z}\Bigg\}\Bigg|=$$

$$=\left|\frac{1}{2\pi i}\int\limits_{\partial D\setminus S}\sum_{k=0}^{n-1}\left(\frac{\left(\overline{z}-\overline{\zeta}\right)^k}{k!}\frac{\partial^k W(\zeta)}{\partial\overline{\zeta}^k}\right)\exp\left[\sigma\left\{\lambda(\zeta)-\lambda(z)\right\}\right]\frac{\exp\left[\sigma\left\{\lambda(\zeta)-\lambda(z)\right\}\right]d\zeta}{\zeta-z}\right|\le$$

$$\le\frac{1}{2\pi}C_1\left(\sum_{k=0}^{n-1}M_k\frac{2^k}{k!}\right)\exp\left[-\sigma\omega(z,\theta',\theta'')\right]=C\exp\left[-\sigma\omega(z,\theta',\theta'')\right]$$

(7)

tengsizlikni hosil qilamiz, bu yerda

$$C=\frac{1}{2\pi}C_1\sum_{k=0}^{n-1}M_k\frac{2^k}{k!},\ C_1=\int_{\partial D}\frac{|d\zeta|}{|\zeta-z|},\qquad M_k=\max_{\zeta\in\partial D\setminus S}\left|\frac{\partial^k W(\zeta)}{\partial\bar{\zeta}^k}\right|$$

(7) dan (3) formulaning o'rinli ekanligi kelib chiqadi.

(3) va (4) formulalar yordamida yuqorida aytib o'tilgan davom ettirish masalasi yechimining kriteriyasini olish mumkin. Buning uchun $D_1=D\cup\{z:\theta'<\arg z<\theta''\}$ sohani qaraymiz.

2-**teorema**. $f\in L_p(D_1),\ p>2$ *va* $\varphi_k\in L(S)\cap C^{n-k-1}(S^\circ),\ (k=0,\ldots,n-1)$

$S^0=\mathrm{Int}S$ *bo'lsin. U holda:*

1) *Agar* D *sohada (1) tenglamani va*

$$W(z)=\varphi_0(z),\ \frac{\partial W(z)}{\partial\bar{z}}=\varphi_1(z),\ldots,\frac{\partial^{n-1}W(z)}{\partial\bar{z}^{n-1}}=\varphi_{n-1}(z),z\in S \tag{8}$$

chegaraviy shartlarni qanoatlantiruvchi $W(z)\in C^n(D)\cap C^{n-1}(\overline{D})$ *funksiya mavjud bo'lsa*

$$\left|\int_0^\infty\left[\frac{1}{2\pi i}\int_S exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\frac{\lambda(\zeta)-\lambda(z)}{\zeta-z}\right.\right.$$

$$\left[\varphi_0(\zeta)+\left(\bar{z}-\bar{\zeta}\right)\varphi_1(\zeta)+\ldots+\frac{\left(\bar{z}-\bar{\zeta}\right)^{n-1}\varphi_{n-1}(\zeta)}{(n-1)!}\right]d\zeta-$$

$$\left.\left.-\frac{1}{\pi}\iint_D\exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\frac{\left(\bar{z}-\bar{\zeta}\right)^{n-1}}{(n-1)!}f(\zeta)\frac{\lambda(\zeta)-\lambda(z)}{\zeta-z}d\xi d\eta\right]d\sigma\right|<\infty \tag{9}$$

xosmas integral har bir $z\in D$ uchun yaqinlashadi *(*$K\subset D_1$ *kompaktlarda tekis yaqinlashadi).*

2) *agar* $\varphi_0(\zeta),\varphi_1(\zeta),\ldots,\varphi_{n-1}(\zeta)$ va $f(\zeta)$ *funksiyalar (9) shartni qanoatlantirsa, u holda (1) tenglamaning* D *sohada o'zining* $(n-1)$ *-tartibgacha hosilalari bilan* S° *gacha uzluksiz bo'lgan va* S *da* (8) *shartlarni qanoatlanuvchitir* $W(z)$ *yechimi mavjud bo'lib, bu yechim (3) va (4) ekvivalent formulalar bilan beriladi.*

Isbot. 1) $W(z)\in C^n(D)\cap C^{n-1}(\overline{D})$ (1) tenglamaning D sohada yechimi bo'lib, (2) shartlarni qanoatlantirsin. $\varphi_k(\zeta)(k=0,1,\ldots,n-1)$ funksiyalar uchun (9) shartning bajarilishini ko'rsatamiz. Kompleks Grin formulasini qaraymiz:

$$\frac{1}{2i}\int_{\partial D}F(\zeta)d\zeta=\iint_D\frac{\partial F(\zeta)}{\partial\bar{\zeta}}d\zeta$$

Bu formulada

$$F(\zeta)=\frac{1}{\pi}\sum_{k=0}^{n}\frac{\left(\bar{z}-\bar{\zeta}\right)^k}{k!}\frac{\partial^k W(\zeta)}{\partial\bar{\zeta}^k}\frac{\lambda(\zeta)-\lambda(z)}{\zeta-z}\exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]$$

funksiyani qo'yib

$$\frac{1}{2\pi i}\int_S \exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\sum_{k=0}^{n}\left(\frac{(\bar{z}-\bar{\zeta})^k}{k!}\varphi_k(\zeta)\right)\frac{\lambda(\zeta)-\lambda(z)}{\zeta-z}d\zeta -$$

$$-\frac{1}{\pi}\iint_D \frac{(\bar{z}-\bar{\zeta})^{n-1}}{(n-1)!}\exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]f(\zeta)\frac{\{\lambda(\zeta)-\lambda(z)\}}{\zeta-z}\frac{\partial^n W}{\partial\bar{\zeta}^n}d\xi d\eta =$$

$$-\frac{1}{2\pi i}\int_{\partial D\setminus S}\exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\sum_{k=0}^{n}\left(\frac{(\bar{z}-\bar{\zeta})^k}{k!}\frac{\partial^k W(\zeta)}{\partial\bar{\zeta}^k}\right)\frac{\{\lambda(\zeta)-\lambda(z)\}}{\zeta-z}d\zeta$$

Tenglikni olamiz. Bu tenglikdan $K\subset D_1$ kompakt olib $z\in K$ uchun quyidagi tengsizlikni hosil qilamiz:

$$\left|\left[\frac{1}{2\pi i}\int_S exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\sum_{k=0}^{n}\left(\frac{(\bar{z}-\bar{\zeta})^k}{k!}\frac{\partial^k w(\zeta)}{\partial\bar{\zeta}^k}\right)\frac{\lambda(\zeta)-\lambda(z)}{\zeta-z}d\zeta -\right.\right.$$

$$\left.\left.-\frac{1}{\pi}\iint_D \exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\frac{(\bar{z}-\bar{\zeta})^{n-1}}{(n-1)!}f(\zeta)\frac{\lambda(\zeta)-\lambda(z)}{\zeta-z}d\xi d\eta\right]\right| = \qquad (10)$$

$$\left|-\frac{1}{2\pi i}\int_{\partial D\setminus S}\exp\left[\sigma\{\lambda(\zeta)-\lambda(z)\}\right]\sum_{k=0}^{n}\left(\frac{(\bar{z}-\bar{\zeta})^k}{k!}\frac{\partial^k W(\zeta)}{\partial\bar{\zeta}^k}\right)\frac{\{\lambda(\zeta)-\lambda(z)\}}{\zeta-z}d\zeta\right| \le$$

$$\le C_2\exp\left[-\sigma\omega(z,\theta',\theta'')\right]$$

Bu yerda C_2 -o'zgarmas son. (10) dan (9) shartning bajarilishi kelib chiqadi.

2) $\varphi_k\in C^{n-k-1}(S)$, *(k=0,1,..,n-1) va* f *funksiyalar (9) shartni qanoatlantirsin. (1) tenglamaning (2) shartlarini qanoatlantiruvchi* $W\in C^n(D)\cap C^{n-1}\left(D\cup S^0\right)$, *yechimi mavjud bo'lishini ko'rsatamiz.* Buning uchun (4) tenglikni o'ng tomonidagi ifodani (*) orqali belgilaymiz. (*) dagi birinchi va ikkinchi qo'shiluvchilar $W_1(z)$ Borel-Pompey tipidagi integrali bo'lib D sohada (1) tenglamani qanoatlantiradi va $D_1\setminus\overline{D}$ sohada bir jinsli polianalitik tenglamani qanoatlantiradigan $W_2(z)$ funksiyani beradi. Bu funksiyalar va ularning hosilalarining S dagi chegaraviy qiymatlari Saxotskiy-Plemel formulasiga asosan [10]

$$\frac{\partial^k W_1(\zeta)}{\partial \bar{\zeta}^k} - \frac{\partial^k W_2(\zeta)}{\partial \bar{\zeta}^k} = \varphi_k(\zeta), \left(k = 0,1,...,n-1\right) \zeta \in S$$

tengliklarni qanoatlantiradi. (*) dagi uchinchi xosmas integral teorema shartiga ko'ra D_1 sohada polianalitik funksiyani ifodalaydi. Shunday qilib, (*) ifoda D sohada (1) tenglamaning yechimi - $h_1(z)$ funksiyani va $D_1 \setminus \overline{D}$ sohada polianalitik bo'lgan $h_2(z)$ funksiyani aniqlaydi. Bu funksiyalarning va hosilalarining S dagi chegaraviy qiymatlari:

$$\frac{\partial^k h_1(\zeta)}{\partial \bar{\zeta}^k} - \frac{\partial^k h_2(\zeta)}{\partial \bar{\zeta}^k} = \varphi_k(\zeta), \left(k = 0,1,...,n-1\right) \zeta \in S$$

tengliklarni qanoatlantiradi. Ikkinchi tamondan (*) ifoda (3) tenglikning o'ng tomonidagi ifodaga teng. (3) formuladan $\varepsilon > 0$ uchun $\left(D_1 \setminus \bar{D}\right) \cap \left\{z : |z| > 1 + \varepsilon\right\}$ sohada $h_2(z) = 0$ ekanligi kelib chiqadi. Polianalitik funksiyalar uchun yagonalik teoremasiga ko'ra $D_1 \setminus \overline{D}$ sohada $h_2(z) \equiv 0$ ekanligi kelib chiqadi. Bundan $h_2(z)$ funksiyaning (n-1)-tartibgacha hosilalari bilan $(D_1 \setminus \overline{D}) \cup S^0$ ga uzluksiz davom etishligi kelib chiqadi. U holda $h_1\left(z\right)$ funksiya ham hosilalari bilan $D \cup S^0$ ga uzluksiz davom etadi. (11) tenglikdan

$$\frac{\partial^k h_1(\zeta)}{\partial \bar{\zeta}^k} = \varphi_k(\zeta), \left(k = 0,1,...,n-1\right) \zeta \in S$$

tengliklarning o'rinli bo'lishi kelib chiqadi. Oxirgi tengliklardan teoremaning tasdig'i kelib chiqadi, (1) tenglamaning $W(z)$ izlanayotgan yechimi sifatida $h_1(z)$ funksiyani olish mumkin.

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