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CAUCHY PROBLEM FOR A HOPF-TYPE SYSTEM

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Abstract. We consider the Cauchy problem for a one-dimensional homogeneous system of Hopf-type equations arising in a two-fluid medium. Energy dissipation is assumed to occur solely due to the friction coefficient (analogous to Darcy), and the Cauchy data are given as a finite trigonometric Fourier series. Recurrent systems of ordinary differential equations for the amplitudes of the approximations are obtained. A particular solution to the resulting ODE is constructed for $N = 3$.

Keywords: two-phase medium, partial density, Cauchy problem, viscosity, Darcy coefficient, Fourier series, Hopf equation.

Xofp tipidagi tenglamalar sistemasi uchun Koshi masalasi

Annotatsiya. Ikki suyuq muhitda yuzaga keladigan Hopf tipidagi tenglamalarning bir o'lchovli bir xil tizimi uchun Koshi muammosini ko'rib chiqamiz. Energiya tarqalishi faqat ishqalanish koeffitsienti tufayli sodir bo'ladi deb taxmin qilinadi (Darsiga o'xshash) va Koshi ma'lumotlari chekli trigonometrik Furye qatori sifatida berilgan. “Yaqinlashuvlar uchun amplitudalar bo‘yicha oddiy differensial tenglamalarning rekurrent tizimlari olindi. Olingan ODT uchun $N = 3$ holatda xususiy yechim qurildi.”

Kalit so‘zlar: ikki fazali muhit, qisman zichlilik, Koshi muammosi, yopishqoqlik, Darsi koeffitsienti, Furye qatori, Hopf tenglamasi.

Задача Коши для системы типа Хопфа

Аннотация. Рассматривается задача Коши для одномерной однородной системы уравнений типа Хопфа возникающая в двухжидкостной среде. Считается, что диссипация энергии происходит только за счет коэффициента трения (аналога Дарси) и данные Коши заданы в виде конечного тригонометрического ряда Фурье. Получены рекуррентные системы обыкновенных дифференциальных уравнений для амплитуд для N приближений. Построено частное решение полученной ОДУ при $N = 3$.

Ключевые слова: двухфазная среда, парциальная плотность, задача Коши, коэффициент Дарси, уравнение Хопфа.

In recent years, mathematicians have become increasingly interested in problems related to the behavior of solutions of systems of partial differential equations, taking into account kinetic parameters. These problems have arisen from physical applications, mainly from modern hydrodynamics (compressible multiphase fluids with low viscosities). An analogy with the Burgers equation arises, for example, in the study of a weakly nonlinear one-dimensional acoustic wave propagating at a linear speed of sound. In this case, the nonlinear terms in velocity in the Burgers-type system of equations arise from the dependence of the speed of sound on the amplitude of the sound

wave, while the second-derivative terms and velocity differences represent the damping of sound waves associated with energy dissipation. In other words, these terms ensure the continuity of solutions and represent dissipative processes associated with entropy production. These terms, in turn, ensure the non-breaking of waves [1]. The system under consideration is a special case of the equations of two-velocity hydrodynamics in the one-dimensional case [2–5].

The one-dimensional analogue of the Navier–Stokes equations for compressible fluids can be considered the system of Hopf-type equations, which is a system of nonlinear convection-diffusion equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -b(u - \tilde{u}), \qquad (1)$$

$$\frac{\partial \tilde{u}}{\partial t} + \tilde{u}\frac{\partial \tilde{u}}{\partial x} = \varepsilon b(u - \tilde{u}), \qquad (2)$$

where the quantities u and$\tilde{u}$can be regarded as the velocities of subsystems with dimension of velocity $[x]/[t]$, constituting a two-speed continuum with corresponding partial densities ρ and $\tilde{\rho}$, b - a positive constant responsible for friction in the system (analogous to the Darcy coefficient) with the dimension $1/[t]$, $\varepsilon = \frac{\rho}{\tilde{\rho}}$ - dimensionless positive constant.

The system of equations of two-velocity hydrodynamics and the system of Hopf-type equations have much in common. For example, quadratic nonlinearity in u and $\tilde{u}$ terms with an advective term corresponding to the dependence of sound on the amplitude of sound waves and the friction coefficient b [1] in the right-hand sides, responsible for the attenuation of sound waves.

As for the properties of the solutions, they are completely different. For the Hopf equation system with a vanishing coefficient b, both strong (shock waves) and weak discontinuities are formed, while solutions of the two-speed hydrodynamics system do not have such features [6-8].

However, the scope of applicability of this system is by no means limited to the examples given; such systems arise in many problems, which determines its significance.

Statement of the problem

Next, we will consider the Cauchy problems for a Hopf-type system with initial conditions at $t = 0$ [9-12]:

$$u|_{t=0} = \varphi_0(x), \qquad \tilde{u}|_{t=0} = \tilde{\varphi}_0(x), \qquad (3)$$

where $\varphi_0(x)$ и $\tilde{\varphi}_0(x) -$ arbitrary bounded measurable functions in R; $|\varphi_0(x)| \le M_0$, $|\tilde{\varphi}_0(x)| \le M_0$.

Let us give the definition of a generalized solution for system (1), (2). The most general space of generalized solutions in the class of ordinary functions is the space of bounded measurable functions $u(t,x)$, $\tilde{u}(t,x)$ in the strip $\Pi_T = [0,T) \times R$.

Definition [1]. Limited measurable in Π_T functions $u(t,x)$, $v(t,x)$ are called a generalized entropy solution (according to Kruzhkov) of problem (1)-(3) if:

For any constant $k_1, k_2 \in R$ and for any trial functions $\varphi(t,x), \psi(t,x) \in C_0^\infty(\Pi_T)$, $\varphi(t,x) \ge 0, \psi(t,x) \ge 0$, inequalities are satisfied$\int_{\Pi_T}\left\{|u-k_1|\varphi_t + sign(u-k_1)\left[\frac{1}{2}(u^2-k_1^2)\varphi_x + b(u-\tilde{u})\varphi\right]\right\}dxdt \ge 0, \qquad (4)$

$$\int_{\Pi_T}\left\{|v-k_2|\psi_t + sign(v-k_2)\left[\frac{1}{2}(v^2-k_2^2)\psi_x - \varepsilon b(\tilde{u}-u)\psi\right]\right\}dxdt \ge 0. \qquad (5)$$

$u(t,x) \to \varphi_0(x)$, $\tilde{u}(t,x) \to \tilde{\varphi}_0(x)$ at $t \to +0$ in space $L_{1,loc}(R)$, that is for any segment $[a,b]$

$$\lim_{t\to+0}\int_a^b |u(t,x) - \varphi_0(x)|\,dx = 0, \quad \lim_{t\to+0}\int_a^b |\tilde{u}(t,x) - \tilde{\varphi}_0(x)|\,dx = 0.$$

Generalized solution $u(t,x)$, $\tilde{u}(t,x)$ in the sense of integral identity is formulated as follows

$$\int_{\Pi_T}\left\{u\varphi_t+\frac{1}{2}u^2\varphi_x-b(u-\tilde{u})\varphi\right\}dxdt=0, \quad (6)$$

$$\int_{\Pi_T}\left\{\tilde{u}\psi_t+\frac{1}{2}\tilde{u}^2\psi_x+\varepsilon b(u-\tilde{u})\psi\right\}dxdt=0, \quad (7)$$

valid for any trial functions $\varphi(t,x),\psi(t,x)\in C_0^{\infty}(\Pi_T)$, and the initial conditions (3) are satisfied for almost all $x\in R$.

Confirmation . If the functions $u(t,x)$, $\tilde{u}(t,x)$ are a generalized entropy solution in the sense of the Definition of problem (1)-(3), then it is a generalized solution of system (1), (2) in the sense of the integral identity (6), (7).

The proof of the proposition follows from inequalities (4), (5) when $k_1=\pm sup|u(t,x)|$, $k_2=\pm sup|\tilde{u}(t,x)|$ due to the arbitrariness of the functions $\varphi(t,x)\geq 0,\psi(t,x)\geq 0$.

The uniqueness of the generalized solution to problem (1)-(3) follows from the following proposition about the stability of solutions with respect to changes in the initial data in the norm of space L_1:

Theorem 1. Let the functions $u(t,x)$, $\tilde{u}(t,x)$ and $v(t,x)$, $\tilde{v}(t,x)$ are generalized solutions of problem (1)-(3) with initial functions $\varphi_0(x)$, $\tilde{\varphi}_0(x)$ and $\psi_0(x)$, $\tilde{\psi}_0(x)$ accordingly and $|u(t,x)|\leq M$, $|\tilde{u}(t,x)|\leq M$ and $|v(t,x)|\leq M$, $|\tilde{v}(t,x)|\leq M$ almost everywhere in the quadrangle $[0,T_0]\times K_R$, here $K_r=\{x||x|\leq r\}$. Then for almost everyone $t\in[0,T_0]$

$$\int_{-r}^{r}\{\rho_1|u(t,x)-v(t,x)|+\rho_2|\tilde{u}(t,x)-\tilde{v}(t,x)|\}dx\leq$$
$$\leq\int_{-r}^{r}\{\rho_1|\varphi_0(x)-\psi_0(x)|+\rho_2|\tilde{\varphi}_0(x)-\tilde{\psi}_0(x)|\}dx. \quad (8)$$

Proof. Let smooth functions $\tilde{\varphi}(t,x;\tau,y)\geq 0$, $\tilde{\psi}(t,x;\tau,y)\geq 0$ finite in $\Pi_T\times\Pi_T$. Let us multiply both sides of inequality (4) by ρ_1 and (5) on ρ_2, respectively. Let's put it this way $k_1=v(\tau,y)$, $k_2=\tilde{v}(\tau,y)$ and $\varphi=\tilde{\varphi}(t,x;\tau,y)$, $\psi=\tilde{\psi}(t,x;\tau,y)$ at a fixed point (τ,y) (note that the functions $v(\tau,y)$, $\tilde{v}(\tau,y)$ are defined almost everywhere in Π_T) and then integrate over Π_T (по (τ,y)):

$$\int_{\Pi_T}\int_{\Pi_T}\{\rho_1|u(t,x)-v(\tau,y)|\tilde{\varphi}_t+sign(u(t,x)-v(\tau,y))\times$$

$$\times\left[\frac{\rho_1}{2}(u^2(t,x)-v^2(\tau,y))\tilde{\varphi}_x-\rho_1 b(u(t,x)-\tilde{u}(t,x))\tilde{\varphi}\right]\}dxdtdyd\tau\geq 0, \quad (9)$$

$$\int_{\Pi_T}\int_{\Pi_T}\{\rho_2|\tilde{u}(t,x)-\tilde{v}(\tau,y)|\tilde{\psi}_t+sign(\tilde{u}(t,x)(t,x)-\tilde{v}(\tau,y))\times$$

$$\times\left[\frac{\rho_2}{2}(\tilde{u}^2(t,x)-\tilde{v}^2(\tau,y))\tilde{\psi}_x-\rho_1 b(\tilde{u}(t,x)-u(t,x))\tilde{\psi}\right]\}dxdtdyd\tau\geq 0, \quad (10)$$

In a completely similar way, from the integral inequalities (4) multiplied by ρ_1 and (5) multiplied by ρ_2 for functions $\tilde{u}(\tau,y)$, $\tilde{v}(\tau,y)$, recorded in variables (τ,y), at $k_1=u(t,x)$, $k_2=\tilde{u}(t,x)$ and $\varphi=\tilde{\varphi}(t,x;\tau,y)$, $\psi=\tilde{\psi}(t,x;\tau,y)$ after integration over Π_T (over (t,x)):

$$\int_{\Pi_T}\int_{\Pi_T}\{\rho_1|v(\tau,y)-u(t,x)|\tilde{\varphi}_\tau+sign(v(\tau,y)-u(t,x))\times$$
$$\times\left[\frac{\rho_1}{2}(v^2(\tau,y)-u^2(t,x))\tilde{\varphi}_y-\rho_1 b(v(\tau,y)-\tilde{v}(\tau,y))\tilde{\varphi}\right]\}dyd\tau dxdt\geq 0 \quad (11)$$
$$\int_{\Pi_T}\int_{\Pi_T}\{\rho_2|\tilde{v}(\tau,y)-\tilde{u}(t,x)|\tilde{\psi}_\tau+sign(\tilde{v}(\tau,y)-\tilde{u}(t,x))\times$$
$$\times\left[\frac{\rho_2}{2}(\tilde{v}^2(\tau,y)-\tilde{u}^2(t,x))\tilde{\psi}_y-\rho_1 b(v(\tau,y)-\tilde{v}(\tau,y))\tilde{\psi}\right]\}dyd\tau dxdt\geq 0. \quad (12)$$

Let's put it this way $\tilde{\varphi}=\tilde{\psi}$. Adding (9), (10) and (11), (12), we get

$$\int_{\Pi_T}\int_{\Pi_T}\{[\rho_1|u(t,x)-v(\tau,y)|+\rho_2|\tilde{u}(t,x)-\tilde{v}(\tau,y)|](\tilde{\varphi}_t+\tilde{\varphi}_\tau)+$$
$$+\frac{\tilde{\varphi}_x+\tilde{\varphi}_y}{2}[\rho_1\, sign(u(t,x)-v(\tau,y))(u^2(t,x)-v^2(\tau,y))+$$
$$+\rho_2\, sign(\tilde{u}(t,x)-\tilde{v}(\tau,y))(\tilde{u}^2(t,x)-\tilde{v}^2(\tau,y))]\}dxdtdyd\tau\geq 0\,. \qquad (13)$$

Let $\varphi(t,x)$ - an arbitrary test function from the definition such that $\varphi(t,x)\equiv 0$ outside some rectangle$\Pi=\{(t,x)|\tilde{\rho}\leq t\leq T-2\tilde{\rho}, x\in K_{r-2\tilde{\rho}}\},\ 2\tilde{\rho}\leq min(r,T)$.

Let us put in (13)

$\tilde{\varphi}=\varphi\left(\frac{t+\tau}{2},\frac{x+y}{2}\right)\delta_h\left(\frac{t-\tau}{2}\right)\delta_h\left(\frac{x-y}{2}\right)$, $h\leq\tilde{\rho}$, here $\delta_h(t)=h^{-1}\delta(t/h)$, $\delta(t)\in C^\infty(-\infty,\infty)$, which $\delta(t)\geq 0$, $\delta(t)\equiv 0$ at $|t|\geq 1$ and $\int_{-\infty}^{\infty}\delta(t)dt=1$.

Subsequence $\{\delta_h(t)\}$ is a delta-shaped sequence at the point $t=0$ at $h\to+0$.

Direct calculations show the validity of the following relationships:

$\tilde{\varphi}_t+\tilde{\varphi}_\tau=\varphi_t\delta_h\left(\frac{t-\tau}{2}\right)\delta_h\left(\frac{x-y}{2}\right)$,

$\tilde{\varphi}_x+\tilde{\varphi}_y=\varphi_x\delta_h\left(\frac{t-\tau}{2}\right)\delta_h\left(\frac{x-y}{2}\right)$.

Repeating the reasoning from [14], from (13) we obtain

$$\int_{\Pi_T}\{[\rho_1|u(t,x)-v(t,x)|+\rho_2|\tilde{u}(t,x)-\tilde{v}(t,x)|]\varphi_t(t,x)+$$
$$+\frac{\varphi_x(t,x)}{2}[\rho_1\, sign(u(t,x)-v(t,x))(u^2(t,x)-v^2(t,x))+$$
$$+\rho_2\, sign(\tilde{u}(t,x)-\tilde{v}(t,x))(\tilde{u}^2(t,x)-\tilde{v}^2(t,x))]\}dxdt\geq 0, \qquad (14)$$

Let $\Sigma_{u,v}$ and $\Sigma_{\tilde{u},\tilde{v}}$ – sets of measure zero on $[0,T]$ from the definition of a generalized solution (see requirement b) for functions u,v and $\tilde{u},\tilde{v}$ respectively. Let us denote by Σ_μ set of points on $[0,T]$, that are not Lebesgue points of a bounded measurable function

$\mu(t)=\int_{-r}^{r}[\rho_1|u(t,x)-v(t,x)|+\rho_2|\tilde{u}(t,x)-\tilde{v}(t,x)|]dx$.

Let $\Sigma_0=\Sigma_\mu\cup\Sigma_{u,v}\cup\Sigma_{\tilde{u},\tilde{v}}$. It is clear that $mes\,\Sigma_0=0$. Repeating the reasoning from [14], we obtain

$$\int_0^{T_0}\left\{[\delta_h(t-\tilde{\rho})-\delta_h(t-\tilde{\tau})]\int_{-r}^{r}[\rho_1|u(t,x)-\tilde{u}(t,x)|+\rho_2|v(t,x)-\tilde{v}(t,x)|]dx\right\}dt\geq 0\quad, \qquad (15)$$

here $\tilde{\rho},\tau\in(0,T_0)\backslash\Sigma_0$, $\tilde{\rho}<\tau$.

Because $\tilde{\rho}$ and τ - Lebesgue points of a function $\mu(t)$, then from (15) at $h\to 0$ we will receive

$$\mu(\tau)=\int_{-r}^{r}[\rho_1|u(\tau,x)-v(\tau,x)|+\rho_2|\tilde{u}(\tau,x)-\tilde{v}(\tau,x)|]dx\leq$$
$$\leq\int_{-r}^{r}[\rho_1|u(\tilde{\rho},x)-v(\tilde{\rho},x)|+\rho_2|\tilde{u}(\tilde{\rho},x)-\tilde{v}(\tilde{\rho},x)|]dx=\mu(\tilde{\rho}). \qquad (16)$$

Considering that

$|u(\tilde{\rho},x)-v(\tilde{\rho},x)|\leq|u(\tilde{\rho},x)-\varphi_0(x)|+|v(\tilde{\rho},x)-\psi_0(x)|+|\varphi_0(x)-\psi_0(x)|$,

directing $\tilde{\rho}$ to zero along a certain sequence of points belonging Σ_0, from (16) taking into account

$$\int_{-r}^{r}\{\rho_1|u(t,x)-v(t,x)|+\rho_2|\tilde{u}(t,x)-\tilde{v}(t,x)|\}dx\leq$$
$$\leq\int_{-r}^{r}\{\rho_1|\varphi_0(x)-\psi_0(x)|+\rho_2|\tilde{\varphi}_0(x)-\tilde{\psi}_0(x)|\}dx.$$

we obtain the estimate (8). From Theorem 1 it follows that:

Theorem 2. Generalized solution of problem (1)-(3) in the layer Π_T only.

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