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PLANE-RADIAL ANOMALOUS FILTRATION OF LIQUID IN HOMOGENEOUS POROUS MEDIUM

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Abstract. The problem of anomalous liquid filtration in a plane-radial porous medium with fractal structure is considered. The process is described by the piezoconductivity equation on derived from the generalized Darcy's law including the local fractional derivative of the pressure gradient. A one-dimensional plane-radial filtration problem has been developed and numerically solved. The problem is solved by the finite difference method. The pressure and filtration velocity fields were determined and analyzed based on the results of calculations.

Keywords: Anomalous filtration, anomalous Darcy's law, filtration velocity, fractional derivative, plane-radial filtration, pressure.

Birjinsli tekis-radial muhitda anomal filtratsiya

Annotatsiya. Fraktal tuzilishga ega tekis-radial g‘ovak muhitda suyuqlikni anomal sizish masalasi qaralgan. Jarayon bosim gradientining kasr hosilasini o‘z ichiga olgan umumlashtirilgan Darsi qonunidan olingan bosimo'tkazuvchanlik tenglamasi bilan tavsiflanadi. Tekis-radial muhitda sizishning bir o‘lchovli masalasi ishlab chiqilgan va sonli yechilgan. Masala chekli ayirmalar usuli yordamida yechilgan. Hisoblash natijalariga ko‘ra, bosim va sizish tezligi maydonlari aniqlangan va tahlil qilingan.

Kalit so‘zlar: anomal filtratsiya, anomal Darsiy qonuni, bosim, kasr hosila, sizish tezligi, tekis-radial sizish.

Плоскопараллельная аномальная фильтрация в однородной среде

Аннотация. Рассматривается задача аномальной фильтрации жидкости в плоскорадиальной пористой среде с фрактальной структурой. Процесс описывается уравнением пьезопроводности, выведенным из обобщенного закона Дарси, включающего локальную дробную производную градиента давления. Была поставлена и численно решена одномерная задача плоскорадиальной фильтрации. Задача решена методом конечных разностей. По результатам численных расчетов определены и проанализированы поля давления и скорости фильтрации. Установлено влияние аномальности на изменение фильтрационных характеристик.

Ключевые слова: аномальная фильтрация, аномальный закон Дарси, давление, дробная производная, скорость фильтрации, плоскорадиальная фильтрация.

Introduction. The equation relating the motion of fluid flow through a porous medium to the pressure gradient was formulated by Darcy in the mid-1800s. Darcy's law states that flow is directly proportional to the pressure gradient [1,2]. Various modifications and additions to the filtration law have been proposed to account for processes that do not obey Darcy's law [3,4]. There have been many studies of fluid diffusion in porous media [5,6,7], where different approaches are used to create new models linking pressure gradient and filtration velocities. It has been found that in some cases, the fluid reacts chemically with the porous medium, leading to changes in the pores [7]. Similarly, solids in liquids can be trapped in pores, resulting in a change in permeability over time [6].

The passage of fluids through porous media can cause a change in permeability. To adequately study these phenomena, the classical Darcy equation was modified by introducing the concept of memory, and it was shown that the concept of memory is expressed by a fractional derivative [7-9].

Memory-based models have been developed using fractional derivatives. However, these models have not been sufficiently investigated yet. Some numerical results of realization of filtering models with memory are given in [10,11].

In [12], a two-dimensional problem of matter transport in a cylindrical porous medium was considered. The mathematical model used to represent the process is expressed by a fractional order differential equation. The problem is solved numerically with appropriate initial and boundary conditions. The results were presented graphically and the effect of the fractional order of the derivative on the transfer characteristics was evaluated.

This paper studies the anomalous filtration of a homogeneous fluid in a cylindrical porous medium. Using the apparatus of fractional derivatives, a mathematical model of anomalous filtration in a radial reservoir is constructed. For the piezoconductivity equation, the problem is considered under conditions of constant pressure at the boundary. The problem is approximated numerically using Caputo's definition of fractional derivative [7]. The effect of changing the order of the fractional derivative in the mathematical model on the pressure and filtration velocity profiles is evaluated.

Statement of the problem. The equation of filtration in a radial reservoir in the one-dimensional case has the form:

$$\frac{\partial p}{\partial t} = \chi \frac{1\partial}{r\partial r}\left[r \frac{\partial}{\partial r}\left(\frac{\partial^{\alpha} p}{\partial t^{\alpha}}\right)\right], \tag{1}$$

where $\chi = \frac{k_f}{\mu\beta}$ is the coefficient of piezoconductivity, k_f is the fractal permeability, μ is the dynamic viscosity coefficient, β is the coefficient of elasticity of the medium, $\alpha(0 < \alpha \leq 1)$ is the order of the derivative, p is pressure.

For this equation, it is sufficient to set one initial and two boundary conditions.

Initial condition:

$$p(0, r) = p_0, p_0 = \text{const.} \tag{2}$$

The boundary conditions are as follows:

$$p(t, r_w) = p_w, \ \ p_w = \text{const} \tag{3}$$

$$p(t, \infty) = p_0, p_w < p_0, \tag{4}$$

where r_w is well radius.

Numerical solution algorithm. Equation (1) under conditions (2) and (3), (4), is solved by the finite analogy method [13-15]. For this purpose, we introduce a grid $\bar{\omega}_h = \{r = r_w + i \cdot h, i = 0,1,\ldots,\}$, $\bar{\omega}_\tau = \{t_j = j \cdot \tau, j = 0,1,2,\ldots, M, \tau = T_{\max}/T\}$ in the region $\{0 \leq t \leq T_{\max}, r_w \leq r < \infty\}$, where h is the grid spacing in the direction r, τ is the grid spacing in time. From these two meshes we construct $\bar{\omega} = \bar{\omega}_h \times \bar{\omega}_\tau$ a two-dimensional mesh. Its nodes consist of points $(t_j, r_i), i = 0,1,\ldots,; j = 0,1,2,\ldots,$ M.

On this grid, equation (1) is approximated as

$$\frac{p_i^{j+1}-p_i^j}{\tau} = \chi\frac{\tau^{1-\alpha}}{r_i\cdot h^2\cdot\Gamma(2-\alpha)}$$
$$-\left(r_{i-1/2}+r_{i+1/2}\right)\cdot(S_2)_i^j + r_{i+1/2}\cdot\frac{p_{i+1}^{j+1}-p_{i+1}^j}{\tau} + r_{i+1/2}\cdot(S_3)_i^j, \tag{5}$$

$i=\overline{1,N-1}, j=\overline{0,M-1},$

where

$$(S_1)_i^j = \sum_{k=0}^{j-1}\left(p_{i-1}^{k+1}-p_{i-1}^k\right)\cdot,$$
$$(S_2)_i^j = \sum_{k=0}^{j-1}\left(p_i^{k+1}-p_i^k\right)\cdot,$$
$$(S_3)_i^j = \sum_{k=0}^{j-1}\left(p_{i+1}^{k+1}-p_{i+1}^k\right)\cdot.$$
$$\left(k_p\right)_i = \frac{\tau^{1-\alpha}\chi}{\Gamma(2-\alpha)r_i h^2}.$$

$\Gamma(\ \ )$ is gamma function, $r_{i-1/2}=\frac{r_{i-1}+r_i}{2}$, $r_{i+1/2}=\frac{r_{i+1}+r_i}{2}$, N is a sufficiently large integer, chosen so that r_N enters the region of pressure variation for a given time t_j, p_i^j is the grid function defined at point $\left(t_j,r_i\right)$.

Then equation (5) takes the following form

$$\mathrm{A}p_{i+1}^{j+1} - \mathrm{B}p_i^{j+1} + \mathrm{C}p_{i-1}^{j+1} = -F_i^j,\ i=\overline{1,N-1}, j=\overline{0,M-1}, \tag{6}$$

where

$$A_i = r_{i-1/2}\left(k_p\right)_i;\ \ B_i = \left(k_p\right)_i\left(r_{i-1/2}+r_{i+1/2}\right)+1;\ \ C_i = r_{i+1/2}\left(k_p\right)_i;$$
$$F_i^j = p_i^j + \left(k_p\right)$$
$$-\left(r_{i-1/2}+r_{i+1/2}\right)(S_2)_i^j + r_{i+1/2}(S_3)_i^j.$$

The initial condition is approximated as

$$p_i^0 = p_0,\ \ i=\overline{0,N}. \tag{7}$$

Boundary conditions (3) - (4) in the difference form have the form

$$p_0^{j+1} = p_c, p_N^{j+1} = p_0,\ \ j=0,1,\ldots,T-1. \tag{8}$$

The system of equations (6) is solved by the Tomas algorithm. Let us use the following relation

$$p_i^{j+1} = \delta_{i+1}\cdot p_{i+1}^{j+1} + \eta_{i+1}, \tag{9}$$

where δ_{i+1},η_{i+1} are coefficients of Tomas algorithm.

Using (9) from (6) we obtain the following recurrence formulas for determining the coefficients δ_{i+1},η_{i+1}

$$\delta_{i+1} = \frac{C}{B-A\cdot\delta_i},\ \ \eta_{i+1} = \frac{F+A\cdot\eta_i}{B-A\cdot\delta_i}. \tag{10}$$

The starting values the coefficients of Tomas algorithm α_{i+1},β_{i+1} are determined on the basis of condition (8)

$\delta_1 = 0, \eta_1 = p_w.$

The filtration velocity is determined on the basis of Darcy's relaxation law [7]

$$v = -\frac{k_f}{\mu}\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial p}{\partial r}\right). \tag{11}$$

The grid analog of this relation has the form

$$v_i^{j+1} = -\frac{k_f}{\mu}\frac{\tau^{1-\alpha}}{\mathrm{h}\Gamma(2-\alpha)}\cdot\left((S_1)_i^j + \frac{p_{i+1}^{j+1}-p_{i+1}^j}{\tau} - (S_2)_i^j - \frac{p_i^{j+1}-p_i^j}{\tau}\right). \tag{12}$$

Results and discussion. Some results of calculations for different parameters of the model are presented in Fig.1-3. The following parameter values are used: $k_f = 10^{-13} m^2 \cdot s^{\alpha}$, $\mu = 10^{-2}\text{Pa} \cdot s$, $\beta = 10^{-10}\text{Pa}^{-1}$, $p_0 = 200 \cdot 10^5\text{Pa}$, $p_w = 150 \cdot 10^5\text{Pa}$. Grid parameters: $h = 0.1m$, $\tau = 1s$.

Fig.1 shows graphs of pressure p change at different moments of time for different values of parameter α. As can be seen from these figures with increasing t the pressure in the medium decreases, which is quite natural. For the classical case $\alpha = 0$ (Fig. 1 a), this decrease is most significant. The more intense the pressure drop, the deeper into the reservoir the pressure change propagates. Thus, for $\alpha = 0$at $t = 3600s$ the pressure drop reaches $\sim 50m$. As the value of α increases, the velocity of pressure drop in the medium slows down and the zone of pressure drop propagation decreases (Fig. 1 b, c, d). Thus, at $t = 3600s$, $r - r_w = 5m$ we approximately have the following pressure values: $\sim 186 \cdot 10^5\text{Pa}(\alpha = 0)$, $\sim 190 \cdot 10^5\text{Pa}(\alpha = 0{,}2)$, $\sim 195 \cdot 10^5\text{Pa}(\alpha = 0{,}4)$, $\sim 198 \cdot 10^5\text{Pa}(\alpha = 0{,}6)$. As can be seen from these data, at $\alpha = 0.6$ the pressure decrease becomes insignificant, the pressure is close to the initial p_0. From the graphs (Fig.1), reproductions are made separately for two values of time t for different values of α (Fig.2). From these figures one can clearly trace the influence of α on the pressure distribution in the medium.

Fig.3. shows graphs of change of filtration velocity v at the same parameters as in Fig.1. As can be seen from the presented graphs, as the time t increases, the values of v increase at the corresponding points of r. As α increases, the values of v decrease significantly. The propagation zone of v also decreases with increasing α. Negative values of v mean that the direction of motion of the fluid is opposite to the direction of the coordinate axis r.

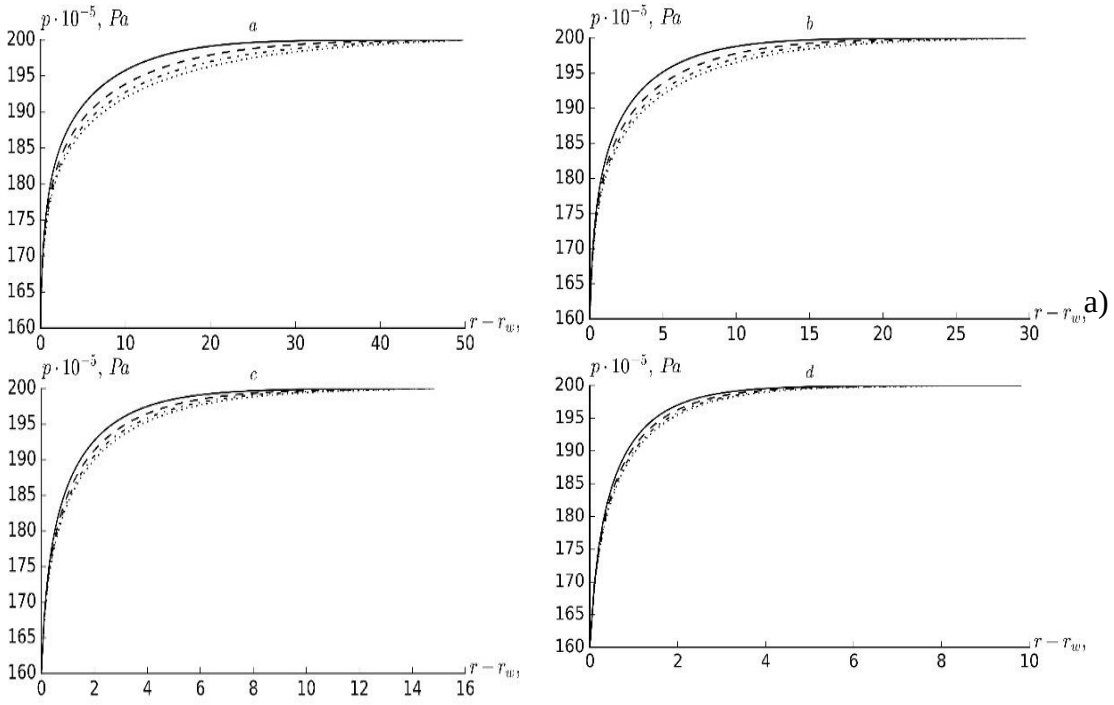

FIGURE 1. Pressure profiles at $\alpha = 0(a)$, $0.2(b)$, $0.4(c)$, $0.6(d)$, $t = 900($ —— $)$, $1800($ - - - $)$, $2700(- - -)$, 3600 s.

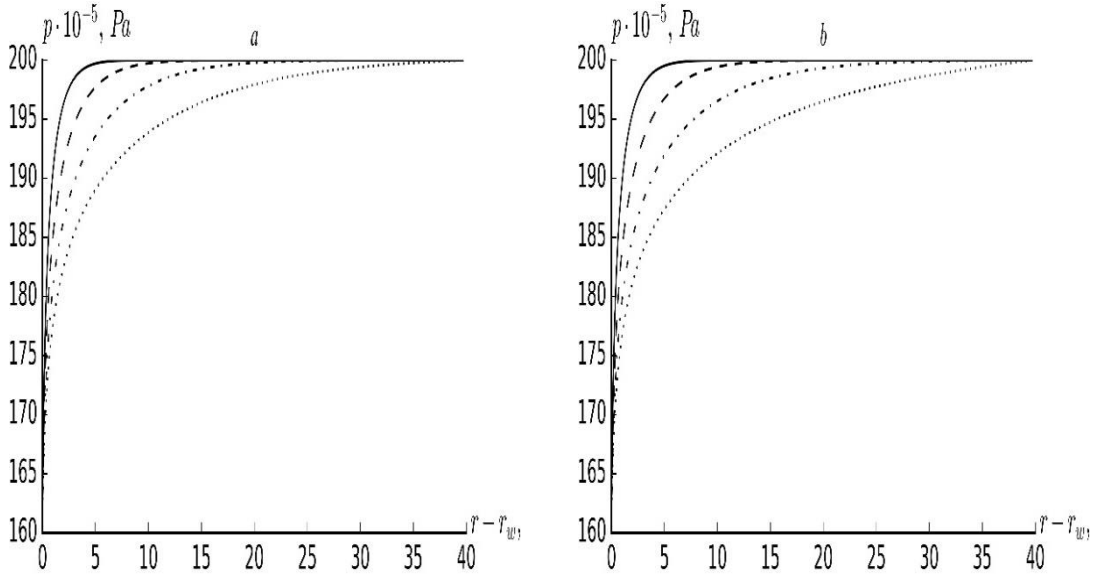

FIGURE 2. Pressure profiles for different αat $t = 1800$(a), 3600 (b) s, $\alpha = 0$, $0.2(- \cdot - -)$, 0.4(- - -), 0.6(——).

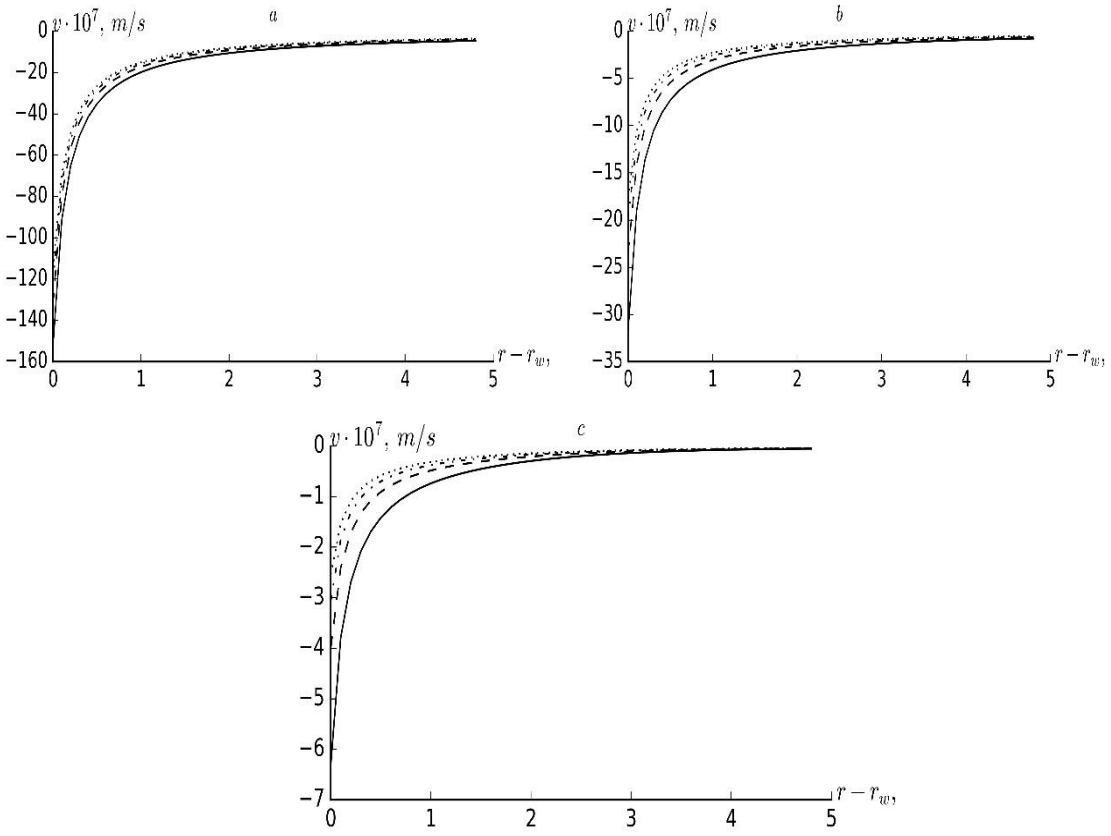

FIGURE 3. Profiles of filtration velocity $\alpha = 0.2(a)$, $0.4(b)$, $0.6(c)$, $t = 900$(——), 1800(- - -), $2700(- - -)$, 3600s.

Conclusions. Anomalous fluid filtration in an unbounded one-dimensional planar-radial porous medium has been analysed. Darcy's law containing the local fractional time derivative of the pressure was used. The piezoconductivity equation in the radial coordinate system is written. A problem is posed for this equation, which is solved by the finite difference method using Caputo's definition for the fractional derivative. It is shown that increasing the order of the fractional derivative leads to a decrease in the velocity of pressure propagation. The velocity of change of filtration velocity decreases similarly. This situation is explained by the phenomenon of delay in the filtration process due to a decrease in permeability [7].

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