ISSN 2181-1296

ILMIY AXBOROTNOMA

НАУЧНЫЙ ВЕСТНИК **SCIENTIFIC JOURNAL**

2021-yil, 1-son (125) **ANIQ FANLAR SERIYASI**

Matematika, Mexanika, Informatika, Fizika

Samarqand viloyat matbuot boshqarmasida ro‘yxatdan o‘tish tartibi 09-25.
Jurnal 1999-yildan chop qilina boshlagan va OAK ro‘yxatiga kiritilgan.

Obuna indeksi – yakka tartbidagi obunachilar uchun **- 5583,**
tashkilot, korxonalar uchun **- 5584**

MUNDARIJA/СОДЕРЖАНИЕ/CONTENTS

UDC: 539.37

STRESS-STREIN STATE OF THE ROCK MASS AROUND THE VERTICAL MINE

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Abstract. The article deals with the problem of the stress-strain state of a rock mass around a vertical working of a circular cross-section. An exact formulation of the three-dimensional problem of the deformation of a half-space weakened by a deep cylindrical cavity is used. The stress-strain state of a half-space, as a three-dimensional body, strictly obeys the basic requirements of the three-dimensional linear theory of elasticity and is described by its corresponding equations and relations in a cylindrical coordinate system. The specific problem of rock mechanics has been solved, i.e. the considered rock mass works only in compression. The deformation process and stress state around vertical shafts of circular cross-section are expressed in terms of stress functions. Calculation formulas are derived for all nonzero components of the strain and stress tensors, taking into account the axisymmetry of the problem under consideration, represented in terms of stress functions.

Keywords: rock mass, vertical working, shaft of vertical mines, stress fields, deformation process, stress function.

Vertikal shaxta atrofidagi tog` jinslari massivining kuchlangan-deformatsiyalangan holati

Annotatsiya. Maqolada doiraviy ko`ndalang kesimli vertikal shaxta atrofidagi tog` jinslari massivining kuchlangan-deformatsiyalangan holati muammosi o'rganilgan. Buning uchun chuqur silindrik bo`shliq bilan kuchsizlantirilgan yarim fazo deformatsiyasi haqidagi masalaning aniq uch o`lchovli qo`yilishidan foydalanamiz. Bunda yarim fazoning kuchlangan-deformatsiyalangan holatini uch o`lchovli jism sifatida, elastiklik chiziqli nazariyasining asosiy talablariga bo`ysinadi va bu nazariyaning tegishli tenglamalari va munosabatlari bilan tavsiflanadi deb hisoblaymiz. Shuning uchun qaralayotgan tog` jinslari massivi faqat siqilishga ishlash sharti bilan qarayotgan masala tog` jinslari mexanikasining aniq masalasini yechishga ishlaydi. Doiraviy ko`ndalang kesimli vertikal shaxta atrofidagi kuchlangan-deformatsiyalangan holatida ko`chishlar va kuchlanishlar tenzorining barcha komponentalarini kuchlanish funksiyasi orqali ifodalanadi. Hisoblash formulalari, ko'rib chiqilayotgan masalaning o`qqa simetrikligini hisobga olgan holda, ko`chish va kuchlanish tenzorlarining barcha nolga teng bo'lmagan tarkibiy qismlari uchun keltirib chiqarilgan.

**Kalit so`zlar:** Tog` jinslari massivi, vertikal shaxta, stvol, kuchlanishlar maydoni, deformatsiya, kuchlanish funksiyasi.

Напряженно-деформированное состояние массива пород вокруг вертикальной шахты

Аннотация. В статье рассмотрена задача о напряженно-деформированном состоянии массива горных пород вокруг вертикальной выработки кругового поперечного сечения. Использована точная постановка трехмерной задачи о деформации полупространства, ослабленной глубокой цилиндрической полостью. Считается, что напряженно-деформированное состояние полупространства, как трехмерного тела, строго подчиняется основным соотношениям трехмерной линейной теории упругости и описывается её соответствующими уравнениями и соотношениями в цилиндрической системе координат. Решена конкретная задача механики горных пород, при условии, что рассматриваемый массив горных пород работает только на сжатие. Компоненты деформации и напряженного состояния вокруг стволов вертикальных шахт кругового сечения выражены через функции напряжения. Выведены формулы вычисления для всех отличных от нуля компонент тензоров деформаций и напряжений, с учетом осесимметричности рассматриваемой задачи.

Ключевые слова: массив горных пород, вертикальная шахта, ствол, поле напряжений, деформация, функция напряжений.

1. Introduction. Real rocks, especially in the conditions of their natural occurrence, exhibit elastic [1], plastic [2] and viscous [3] properties. At the same time, according to the data of a number of authors [4,5,6], even strong rocks with ultimate strength in uniaxial compression $\sigma_{сж} = 8 \cdot 10^3 \div 12 \cdot 10^3\ Па,$ show a significant nonlinearity of the relationship between stresses and deformations even at very low values of the acting stresses [7].

Determination of the parameters of stress fields around workings, taking into account all the features of rock deformation, is a very, very difficult task in mathematical terms [8,9]. In this regard, considering the properties of real rock masses, the main features of their deformation are established and, depending on this, a model of elastic, elastoplastic and viscoelastoplastic medium is used [10, 11]. For rock massifs with high ultimate strength of rocks and high values of elastic characteristics - elastic modulus E and shear deformation coefficient v (Poisson's ratio) - as a rule, sufficient calculation accuracy is ensured when the rocks are endowed with the properties of an ideally elastic medium [12,13].

On the other hand, the use of an ideal elastic model for determining the parameters of stress and strain fields that form immediately after the formation of workings is also natural for massifs composed of less strong and less elastic rocks, since the rate of stress and strain redistribution, as already indicated, is very high and therefore, the plastic and viscous properties of the massif in the first moments of time practically do not have time to be realized. As a consequence, elastic solutions can be considered as the upper limitpossible voltage values in real conditions.

Taking into account the above considerations, let us consider the problem of the stress-strain state (SSS) of a rock mass around a vertical working of a circular cross-section. We will proceed from the exact formulation of the three-dimensional problem of the deformation of a half-space weakened by a deep cylindrical cavity [14,15]. In this case, we will assume that the stress-strain state of a half-space, as a three-dimensional body, strictly obeys the basic requirements of the three-dimensional linear theory of elasticity and is described by its corresponding equations and relations [16-18].

2. Statement of the general problem. Basic equations and relations. To solve the problem, let us refer the space around the working to the cylindrical coordinate system, (r,θ,z), the origin of which is located on the day time surface of the massif, and the z axis coincides with the excavation axis and is directed downward. We will denote $U_r,\ U_\theta,\ U_z$ by moving the points of the array in the direction of the axes $r,\ \theta,\ z$; through $\varepsilon_{rr},\ \varepsilon_{\theta\theta},\ \varepsilon_{zz},\ \varepsilon_{rz},\ \varepsilon_{r\theta},\ \varepsilon_{\theta z}$ – the components of the strain tensor in the coordinate system and through $\sigma_{rr},\ \sigma_{\theta\theta},\ \sigma_{zz},\ \sigma_{rz},\ \sigma_{r\theta},\ \sigma_{\theta z}$ the components of the stress tensor in the same coordinate system.

To determine all components of the stress tensor and the displacement vector in the problem, i.e.to solve the formulated problem, it will be necessary to integrate the three-dimensional equations of elastic equilibrium

$$\sigma_{ij,j}=0, \qquad (i,j=r,\theta,z). \tag{1}$$

Let us choose the statics equations in the Lame form from the many forms of writing these equations. The use of the indicated equilibrium equations will be much easier if we take into account that the components of the rotation vector – $\omega_r,\ \omega_\theta,\ \omega_z$ are related to displacements U_r, U_z, U_θ by the following formulas.

$$\omega_r=\frac{1}{2}\left(\frac{1}{2}\frac{\partial U_z}{\partial\theta}-\frac{\partial U_\theta}{\partial z}\right),\qquad \omega_\theta=\frac{1}{2}\left(\frac{\partial U_r}{\partial z}-\frac{\partial U_z}{\partial r}\right),\qquad \omega_z=\frac{1}{2}\left[\frac{\partial(rU_\theta)}{\partial r}-\frac{\partial U_r}{\partial\theta}\right]. \tag{2}$$

Transforming the equilibrium equations in the Lamé form, taking into account (2), we arrive at the following more convenient form

$$\begin{aligned}
&\frac{\partial\varepsilon}{\partial r}-\frac{2\mu}{\lambda+2\mu}\left(\frac{1}{r}\frac{\partial\omega_z}{\partial\theta}-\frac{\partial\omega_\theta}{\partial z}\right)=0,\\
&\frac{\partial\varepsilon}{\partial\theta}-\frac{2\mu r}{\lambda+2\mu}\left(\frac{\partial\omega_r}{\partial z}-\frac{\partial\omega_z}{\partial r}\right)=0,\\
&\frac{\partial\varepsilon}{\partial z}-\frac{2\mu}{r(\lambda+2\mu)}\left(\frac{\partial(r\omega_\theta)}{\partial r}-\frac{\partial\omega_z}{\partial\theta}\right)=0,
\end{aligned} \tag{3}$$

where

$\lambda=\dfrac{Ev}{(1+v)(1-2v)};\qquad \mu=\dfrac{E}{2(1+v)}$ – - Lame coefficients; *E* – modulus of elasticity (Young).

In this case, volumetric deformation $-\varepsilon$ through displacement U_r, U_θ, U_z is expressed as follows:

$$\varepsilon = \frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \frac{1}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{\partial U_z}{\partial z}.$$

3. Solving the problems. Since the problem of deformation of a half-space, weakened by an infinitely deep cylindrical working of circular cross-section, is considered, the three-dimensional problem can be reduced to two-dimensional, as an axisymmetric one. For this, it is assumed that the load acting on the roadway is distributed symmetrically about the *0z* axis. Then, the displacements of the points of the array are also distributed symmetrically, i.e. they do not depend on the angular coordinate θ.

$$U_r = U_r(r,z);\ U_\theta = U_\theta(r,z);\ U_z = U_z(r,z).$$

In this case, the equilibrium equations (3) are greatly simplified and take the form

$$\frac{\partial \varepsilon}{\partial r} + \frac{1-2\nu}{1-\nu}\frac{\partial \omega}{\partial z} = 0,\ \ \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rU_\theta)}{\partial r}\right] + \frac{\partial^2 U_\theta}{\partial z^2} = 0,\ \ \frac{\partial \varepsilon}{\partial z} - \frac{1}{r}\frac{1-2\nu}{1-\nu}\frac{\partial (rw)}{\partial r} = 0. \tag{4}$$

Wherein

$$\varepsilon = \frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \frac{\partial U_z}{\partial z}; \qquad \omega = \omega_\theta; \qquad \omega_r = \omega_z = 0.$$

In the obtained equations, displacement U_θ is included only in the second equation, and displacement U_r and U_z is included only in the first and third equations. Therefore, it is possible to separate the task of determining the displacement U_θ from the task of determining the displace ments U_r and U_z. The first problem corresponds to the torsion of a cylindrical layer having a finite thickness, the second - to the case of the deformed state of the rock mass under consideration, which is called the axisymmetric problem. Thus, we come to the conclusion that in the future, to solve the problem, it is sufficient to integrate the equations (five)

$$\frac{\partial \varepsilon}{\partial r} + \nu^* \frac{\partial \omega}{\partial z} = 0; \qquad \frac{\partial \varepsilon}{\partial z} - \frac{\nu^*}{r}\frac{\partial (r\omega)}{\partial r} = 0, \tag{5}$$

where

$$\nu^* = (1-2\nu)/(1-\nu).$$

The solution to system (5) can be obtained in various ways. For example, it can be reduced to finding some auxiliary functions introduced in a certain way - stress functions, having previously expressed the displacements and all components of the stress tensor in terms of these functions. Such functions, which are a solution to an axisymmetric problem, were introduced by various authors in different ways, proceeding from the direction of the problems under consideration.

The problem we are considering about the deformation of a half-space, weakened by deep mining, is focused on solving a specific problem in rock mechanics. Therefore, it is natural to assume that the rock mass under consideration works only for compression. Proceeding from these considerations, following the procedure of, but with some difference from it, corresponding to the essence of the particular problem under consideration, we introduce the first of the stress functions as follows:

$$\frac{\partial \varphi}{\partial z} = -\frac{\mu\, r}{1-\nu}\omega; \qquad \frac{\partial \varphi}{\partial r} = -\frac{r(\lambda + 2\mu)}{2(1-\nu)}\varepsilon, \tag{6}$$

Where $\varphi(r,z) = \varphi$ – some function of the variables r and z.

Substituting the expressions ε and ω into the second equation (5), we make sure that it is fulfilled identically. Substituting (6) into the first equation (5), we obtain

$$\nabla^2 \varphi = 0, \tag{7}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.$$

Consequently, the equations of equilibrium in displacements are satisfied if ε and ω in the form

$$\omega = -\frac{1-\nu}{\mu\, r}\frac{\partial\varphi}{\partial z}; \qquad \varepsilon = -\frac{2(1-\nu)}{r(\lambda+2\mu)}\frac{\partial\varphi}{\partial r}$$

And if the function $\varphi = \varphi(r,z)$ is defined as a solution to equation (7)

Using the expressions for volumetric deformation and rotation - (4) and (2) the above expressions and ω through $\varphi(r,z)$ rewrite them in the following form

$$\frac{1}{r}\frac{\partial(rU_r)}{\partial r} + \frac{\partial U_z}{\partial z} = -\frac{2(1-\nu)}{(\lambda+2\mu)}\frac{\partial\varphi}{\partial r}, \qquad \frac{\partial U_r}{\partial z} - \frac{\partial U_z}{\partial r} = -\frac{2(1-\nu)}{\mu}\frac{1}{r}\frac{\partial\varphi}{\partial z}. \tag{8}$$

Now, let's introduce a new helper function $\phi(r,z)$ like this:

$$\frac{\partial\phi}{\partial r} = -\lambda\frac{\partial(rU_r)}{\partial r} + r(\lambda+2\mu)\frac{\partial U_z}{\partial z}, \qquad \frac{\partial\phi}{\partial z} = \mu\, r\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}\right). \tag{9}$$

Subtracting from the first equation (8) the first equation (9), divided by $\lambda + 2\mu,$ and dividing μ by the second equation (9) and adding with the second equation (8), we obtain a system of differential equations with respect to the function U_r. Integrating these equations, we make sure that the connection between the radial displacements U_r and by the introduced stress functions φ and ϕ has the form

$$U_r = \frac{1}{2\mu\, r}\left[\phi - 2(1-\nu)\varphi\right] \tag{10}$$

From the course of elasticity theory [13] it is known that displacement U_r in an axisymmetric problem must satisfy the equation

$$\left(\nabla^2 - \frac{1}{r^2}\right)^2 U_r = 0. \tag{11}$$

Substituting in (11) the expression for U_r - (10) and performing differentiation, and with this in mind (7), we obtain a biharmanic differential equation that determines the stress function ϕ - (10).

$$\nabla^4\phi = 0, \tag{12}$$

where $\nabla^4 = \nabla^2\nabla^2 = \left(\nabla^2\right)^2$ –Biharmonic operator. Hence it follows that the introduced stress function ϕ must be biharmonic.

Substituting in (9) the values of the displacements - (10), U_r through the functions ϕ and φ – (10), *r* and *z* for the derivatives with respect to the coordinates r and z of the longitudinal displacement U_z we obtain a system of differential equations with respect to the function U_z. Hence, taking into account the relationship between the elastic constants we get finally

$$\frac{\partial U_z}{\partial r} = \frac{1}{2\mu\, r}\frac{\partial}{\partial z}\left[\phi + 2(1-\nu)\varphi\right], \qquad \frac{\partial U_z}{\partial z} = -\frac{1}{2\mu\, r}\frac{\partial}{\partial r}\left(\phi - 2\nu\varphi\right), \tag{10$_1$}$$

where ν - is Poisson's ratio.

Substituting the values of the derivatives of displacement U_z in the integrability condition for these equations, taking into account Eq. (7), we obtain

$$\nabla^2\phi = -2\frac{\partial^2\varphi}{\partial z^2}.$$

The last equation is the condition for the integrability of the above equations with respect to the derivatives $\partial U_z/\partial r$ and $\partial U_z/\partial z.$ This condition will be satisfied if the function ϕ is set in the form

$$\phi = \psi + r\frac{\partial\varphi}{\partial r}, \tag{13}$$

where $\psi = \psi(r,z)$ - is a function of coordinates *r* and *z*, that satisfies the differential equation

$$\nabla^2\psi = 0 \tag{14}$$

Hence it follows that thus, the stress function ϕ is presented in the form (13). The advantage of presentation (13) lies in a more correct description of the deformation process around the shafts of vertical shafts of circular cross-section.

There is another way to select the stress function ϕ, given in [19]. According to it, the function ϕ, can be represented in the form

$$\phi = \psi + z\frac{\partial\varphi}{\partial z} \tag{15}$$

It should be noted that the latter representation ϕ is convenient to use in problems requiring exact fulfillment of conditions only on the edges *z=const*, i.e. upon deformation of bodies such as a layer or a half-space. In those cases when the conditions on cylindrical surfaces play a decisive role, as is the case in the problem under consideration, it is necessary to use formula (13). If it is necessary to fulfill the boundary conditions on mutually orthogonal surfaces $z = const$ and *r = const*, both variants of the general solution of the axisymmetric problem should be used.

In conclusion of the section, we note that the next step in solving the general problem is the representation of all nonzero components of the strain and stress tensors, taking into account the axisymmetry of the problem under consideration. Using the expressions connecting the components of the deformation tensor with displacements in a cylindrical coordinate system (r,θ,z) and under the accepted assumptions regarding displacements, i.e. at we have

$$U_r = U_r(r,z),\ U_\theta = 0, \qquad U_z = U_z(r,z)$$

and

$$\varepsilon_{rr} = \frac{\partial U_r}{\partial r}; \quad \varepsilon_{v\theta} = \frac{U_r}{r}; \quad \varepsilon_{zz} = \frac{\partial U_z}{\partial z}; \quad \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}\right); \quad \varepsilon_{r\theta} = \varepsilon_{\theta z} = 0.$$

Introducing into these equalities instead of displacements U_r and U_z functions ϕ and φ in accordance with expressions (10) and expressions for the derivatives of displacement U_z along the coordinates r and z, we obtain

$$\begin{gathered}\varepsilon_{rr} = \frac{1}{2\mu}\cdot\frac{\partial}{\partial r}\frac{\phi - 2(1-v)\varphi}{r}; \quad \varepsilon_{rz} = \frac{1}{2\mu\ r}\frac{\partial\phi}{\partial z};\\ \varepsilon_{zz} = -\frac{1}{2\mu\ r}\frac{\partial}{\partial r}(\phi - 2v\varphi); \quad \varepsilon_{\theta\theta} = \frac{1}{2\mu\ r^2}\left[\phi - 2(1-v)\varphi\right]\end{gathered} \tag{16}$$

To find the stress components, we use Hooke's law for an isotropic body, taking into account the axisymmetry of the problem [20]

$$\begin{gathered}\sigma_{rr} = \lambda\varepsilon + 2\mu\ \varepsilon_{rr}; \quad \sigma_{\theta\theta} = \lambda\varepsilon + 2\mu\ \varepsilon_{\theta\theta}; \quad \sigma_{r\theta} = 2\mu\varepsilon_{r\theta} = 0;\\ \sigma_{zz} = \lambda\varepsilon + 2\mu\ \varepsilon_{zz}; \quad \sigma_{rz} = 2\mu\ \varepsilon_{rz}. \quad \sigma_{z\theta} = 2\mu\varepsilon_{z\theta} = 0;\end{gathered}$$

wherein

$$\varepsilon = \varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz} = -\frac{2(1-v)}{\lambda + 2\mu}\cdot\frac{1}{r}\frac{\partial\varphi}{\partial r} = -\frac{1-2v}{\mu\ r}\frac{\partial\varphi}{\partial r}. \tag{17}$$

Substituting in the last expressions of Hooke's law the values of deformations according to formulas (16) and the expression for volumetric expansion (17), we obtain

$$\begin{gathered}\sigma_{rr} = \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\phi + \frac{2(1-v)}{r^2}\varphi - \frac{2}{r}\frac{\partial\varphi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\left[\phi - 2(1-v)\varphi\right] - \frac{2}{r}\frac{\partial\varphi}{\partial r};\\ \sigma_{zz} = -\frac{1}{r}\frac{\partial\phi}{\partial r}; \quad \sigma_{rz} = \frac{1}{r}\frac{\partial\phi}{\partial z}; \quad \sigma_{\theta\theta} = \frac{1}{r^2}\left[\phi - 2(1-v)\varphi\right] - \frac{2v}{r}\frac{\partial\varphi}{\partial r}.\end{gathered} \tag{18}$$

Thus, all four components of the stress tensor normal σ_{rr}, σ_{zz}, $\sigma_{\theta\theta}$ and tangential σ_{rz}, as well as nonzero components of the displacement vector – radial U_r and longitudinal U_z are expressed through the introduced stress functions ϕ and φ.

To shorten the entries, you can enter the following designation:

$$\overline{\psi} = \frac{1}{r^2}\left[\phi - 2\psi - 2(1-\nu)\varphi\right]. \quad (19)$$

Taking into account (19), the first equation (18) is transformed as follows:

$$\sigma_{rr} = \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\left[\phi - 2(1-\nu)\varphi\right] - \frac{2}{r}\frac{\partial\varphi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\left[\phi - 2(1-\nu)\varphi - 2\psi + 2\phi + 2\psi - 2\phi\right] -$$

$$-\frac{2}{r}\frac{\partial\varphi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\left[3\phi - 2\psi - 2(1-\nu)\varphi\right] + \frac{2}{r^2}\left[\phi - \left(\psi + r\frac{\partial\varphi}{\partial r}\right)\right].$$

Hence, taking into account (19) and (13), it follows that similarly

$$\sigma_{rr} = \frac{1}{r}\frac{\partial\phi}{\partial r} - \overline{\psi};\ \ \sigma_{\theta\theta} = \psi - \frac{2(1+\nu)}{r}\frac{\partial\varphi}{\partial r};\ \sigma_{zz} = -\frac{1}{r}\frac{\partial\phi}{\partial r};\ \ \sigma_{rz} = \frac{1}{r}\frac{\partial\phi}{\partial z}. \quad (20)$$

Formulas (16) and (20) make it possible to determine the stress-strain state of a half-space with a deep cylindrical cavity of circular cross-section, if solutions of quasi-harmonic equations $\nabla^2\varphi = 0, \quad \nabla^2\psi = 0$ are found under the corresponding boundary conditions specified on the cylindrical surface of the cavity and simulating the problem of the stress-strain state of a rock mass, around the vertical working of a circular cross-section.

4. Conclusions. Thus, the specific problem of rock mechanics has been solved, i.e. the considered rock mass works only for compression. The deformation process and stress state around vertical shaft shafts with circular cross-section are expressed in terms of stress-state functions around vertical shaft shafts with circular cross-section expressed in terms of stress functions. Formulas are obtained that make it possible to unambiguously determine the stress-strain state of a half-space with a deep cylindrical cavity of circular cross-section if solutions of quasi-harmonic equations $\nabla^2\varphi = 0, \quad \nabla^2\psi = 0$ are found under the corresponding boundary conditions specified on the cylindrical surface of the cavity and simulating the problem of stress-strain state of a rock mass around a vertical working circular cross section.

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